Scribe Notes: 1/28/2013
Presenter: Dr. Berthe Y. Choueiry
Scribe: Nate Stender
Topic: Temporal Reasoning (Ch. 12 of Dechter book)

Question by Daniel Dobbs:

How do we know when a constraint is convex?

a. In a CSP, we can look at the bit-matrix representation of the binary constraints:

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

If we can swap rows in the bit-matrix representation such that all of the ones in each row are consecutive (consecutive-ones property), then we say that the constraints are row convex. [1] If the constraints remain row convex after enforcing consistency (a posteriori condition), then a known result guarantees that PC => Minimality => Decomposable.[2]

b. In a CSP where constraints or domains can be arranged as intervals with a total ordering and that those intervals are not fragmented by the operations of enforcing consistency (composition and intersection), then the constraints are convex.

Temporal Reasoning: ΔSTP

The algorithm:

- Is a refinement of the algorithm for PPC[3]
- Works by updating all edges in a triangle at every propagation step
- Changes are propagated to adjacent triangles

Advantages of ΔSTP

- Cheaper than PPC and F-W (empirical proof)
- Guarantees the minimal network
- Automatically decomposes graph into its bi-connected components
    - Binds effort in the size of the largest component

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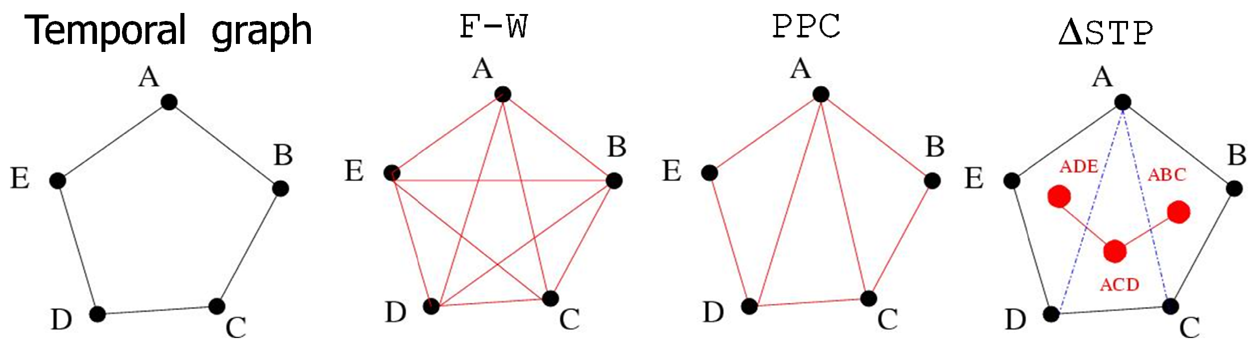
[1] See textbook page 231—237.

[2] On the Minimality and Decomposability of Row-Convex Constraint Networks, Peter van Beek and Rina Dechter, *Journal of the ACM*, Vol 42(3). Pages 543—561. 1995

[3] Path Consistency on Triangulated Constraint Graphs, Christian Bliek and Djamila Sam-Haroud, *Proceedings of IJCAI 1999*. Pages 456—461. 1999.

- Allows for parallelization
- Sweep through back and forth (relation to tree decomposition made by N. Wilson in 2005).



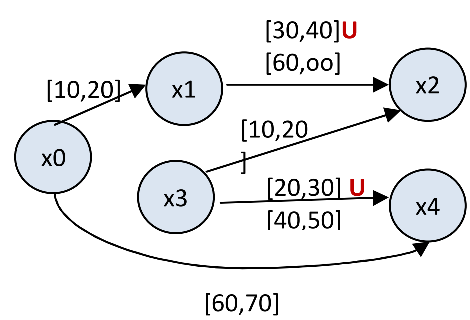
Recent Advances in ΔSTP [4]

- Exploit structure
  - Order variables linearly
  - Use a PEO (bottom-up ordering) or Max Cardinality ordering (top down ordering)
- Apply Directional Path Consistency (Ch 4. Dechter): Determines consistency
- Propagate down: Provides minimal network

Temporal Reasoning: Solving the TCSP

Review of TCSP

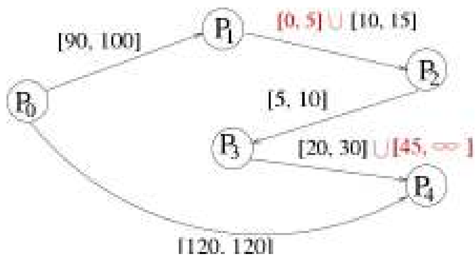
- Variables represent time points, each with a continuous domain
- Each constraint is a disjunction of intervals
- Solution is an assignment which does not violate any constraints
- Deciding consistency of a TCSP instance is NP-complete

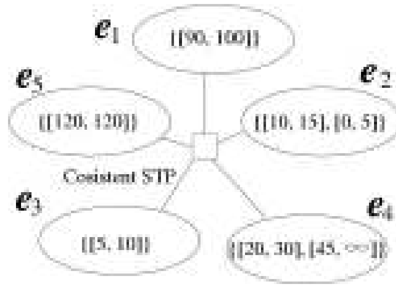


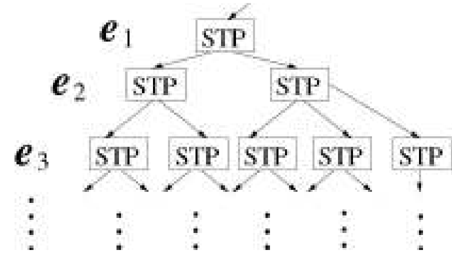
Solving the TCSP

- Formulate TCSP as meta-CSP
- Find all solutions to the meta-CSP
- Use ULT (Dechter's textbook), or ΔSTP to solve the individual STPs efficiently (Xu Lin's CP 2003)

[4] P3C: A New Algorithm for the Simple Temporal Problem, Leon Planken, Mathijs de Weerdt, Roman van der Krogt *Proceedings of ICAPS 2008*, Pages 256—263. 2008.

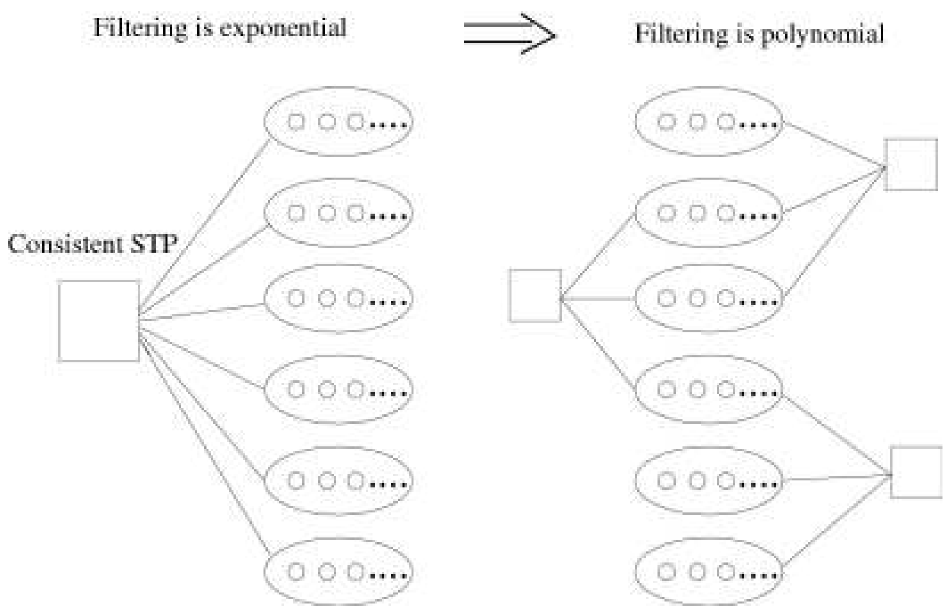






Preprocessing the TCSP

- Arc consistency on the meta-CSP (with GAC) is NP-hard because it corresponds to solving the TCSP
- Use ΔAC to filter the domains of TCSP with ternary constraints
- ΔAC removes values that are not supported in the ternary constraint.



ΔSTP is very effective at low density, ΔAC is very effective at high density. We can combine them to create a powerful tool.