**Topic: Quantitative Temporal Networks**

**Relevant literature:**
- Chapter 12 of Dechter’s Textbook
- *Chapter on Temporal Reasoning* by Manolis Koubarakis in Handbook of Constraint Programming by Rossi et al.
- *Temporal Reasoning Powerpoint Slides by Dr. Berthe Y. Choueiry*

**Introduction**
Quantitative temporal networks is a convenient formalism to express *metric* information as a *distance* between two time points. A time point may represent
- an event or
- the beginning or the ending of some episode.

The duration of time can be expressed as: e-s
- s represents the starting point
- e represents the ending point

Example: John’s travel by car from home to work takes him 30 to 40 minutes or if he travels by bus, it takes him at least 60 minutes
- Time points:
  - $X_1$ John leaves home
  - $X_2$ John arrives at work
- Constraint:
  - $(30 \leq x_2 - x_1 \leq 40)$ or $(60 \leq x_2 - x_1)$

**Main Types of Metric Temporal Constraint Problems: STP, TCSP, DTP**

(Graphs taken from PowerPoint slides by instructor)

**Simple Temporal Problem (STP)**
- Each edge has a unique (convex) interval
- Tractable, can be solved in polynomial time by transforming to a distance graph and then applying Floyd-Warshall or path consistency algorithm to find compute the minimal network. Any solution can be built backtrack free.
Temporal CSP (TCSP)
- Each edge has a disjunction of intervals
- STP ⊆ TCSP
- Deciding consistency of an instance of TCSP is NP Complete
- But we can solve TCSP by transforming to many STPs. The idea is you select one of the intervals in a disjunction of intervals for each edge, and check whether the resulting STP is consistent.

Disjunctive Temporal Problem (DTP)
- Introduced by Stergiou & Koubarakis
- Each constraint is a disjunction of edges
- For each constraint, you have to satisfy at least one of the edges.
- TCSP = DTP (detected by Robert)
- NP Complete
- TCSP is able to represent every DTP.
- Mapping from DTP to TCSP is given in Leon Planken paper.
- Extensive research done on this topic by Martha Pollack (now University of Michigan, Ann Arbor) with application to assisted living.

Simple Temporal Problem (STP)

One important property of STP is that every edge represents a single interval and does not contain any disjunction of intervals. This property makes STP tractable, which means it can be solved in polynomial time. STP can be solved in two equivalent ways:

1. Path Consistency algorithm with only three loops: uses two operators intersection and composition
2. Floyd-Warshall on the distance graph

Let’s consider the example from slides...
- x0 = 7:00am
- x1 John left home between 7:10 to 7:20
- x2 John arrive work in 30 to 40 minutes
- x3 Fred left home 10 to 20 minutes before x2
- x4 Fred arrive work between 8:00 to 8:10
- Fred travel from home to work in 20 to 30 minutes

Query: Is this scenario possible? Did Fred leave home before or after John?

Example 12.7 (page 345)
Solving the STP with Path Consistency

We need to define the two operators....

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Composition</th>
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| The intersection of \( T \) and \( S \) admits only values that are allowed by both \( T \) and \( S \). | \( T = [a,b] \)  
\( S = [c,d] \)  
\( T \setminus S = [\text{maximum}(a,c), \text{minimum}(b,d)] \) |

The application of PC on the example gives the following result

\[ \rightarrow \text{Insert here the example above after the addition of the missing constraints and after tightening all the constraints by path consistency.} \]

Solving the STP using Floyd Warshall

STP is transformed to a distance graph to convert it into all-pairs-shortest-paths problem where we can use Floyd-Warshall to get the minimal distances. We convert the constraint graph of STP to a directed cyclic graph by replacing each constraint by two directed edges, labeled with the upper and lower bounds of the interval (positive and negative labels, respectively).

Example:

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The application of the Floyd Warshall yields the following network:

- **Insert here the example the graph with the shortest distances.**

  - The problem is inconsistent if we find a cycle with a negative distance.
  - The problem is said to be minimal and decomposable if there is no negative cycle after applying Floyd-Warshall algorithm.
  - Time complexity is Floyd-Warshall $O(n^3) +$ Assembling $O(n^3) = O(n^3)$.

**Detecting inconsistency: An example**

- **Insert here an example of an inconsistency detected by PC and by F.-W.**
Scribe Notes: 1/16/2013
Presenter: Dr. Berthe Y. Choueiry
Scribe: Istiaque Ali
Topic: Q&As Section - Temporal Reasoning (Chapter 12 of Dechter book)

Other relevant books and papers:
Temporal Reasoning Powerpoint Slides by Dr. Berthe Y. Choueiry
Handbook of Constraint Programming by Rossi et al.
Triangle-AC by Choueiry and Xu, AI Communications, 2005.
Triangle-STP by Xu and Choueiry, TIME 2003.
TCSP by Xu and Choueiry, CP 2003.
TCSP by Dechter, Meiri & Pearl, AIJ 1991.

Question and Answers Section:

Daniel’s Question

1. Graph Theory Class
   a. CSE 924 Graph Algorithms. Dr. Deogun used to teach a 496/896 graph algorithms class some time ago (Graph Algorithms for non CS Students?).
   b. Math Department offers two Graph Theory classes for undergraduate and another two for graduate students. The class for grad students is tough because it is for preparation for the Math Qualifying Exam. But you may be able to take for P/NP or as an independent study course (more lenient grading).

2. Point algebra is not as expressive as interval algebra (i.e., PA cannot represent all IA problems). For example before and after. We can go around it by using higher arity constraint.
   → We are creating a new algebra by extending the point algebra which has binary constraints to include non-binary constraints. This new algebra has different complexity and different representation than point algebra.

Zion’s Question and Comments

1. TCSP use for macroscopic problems in temporally-grounded planning problems.
   a. STP is used for space mission scheduling.
      i. Program was written in Lisp
      ii. It was done in 1996
      iii. Application was monitoring and scheduling the execution of scientific experiments in space
      iv. It was the Remote Agent on Deep Space 1.
   b. DTP is used in medical application for assistance to elderly people
i. By Martha Pollock at University of Michigan
ii. She had many projects and application which dealt with medical problems and assisted living
   c. TCSP is too expensive to use in hardware problems and scheduling jobs
      i. Solving the TCSP is NP-Complete
      ii. scheduling at the hardware level is usually solved with greedy algorithms and quick heuristics (e.g., starting from deadlines, backward). We want linear or sub-linear algorithms to solve those problems because of the time constraints.
   d. Xerox built expensive copying machines which scheduled print jobs using TCSP. Although it was a hardware problem but it is not as fast as a CPU so they can have disjunctions.

2. How well do TCSP scale as constraints are added badly?
   a. NP-Complete and exponential

3. Temporal Granularity and Temporal Reasoning
   There are various approaches to temporal reasoning in CS and AI, can be distinguished by the formalism used/provided:
      a. Logics: various temporal logics by theoreticians with application to hardware, hybrid systems, software verification
      b. Constraint Networks (CP community)
      c. Temporal databases
      d. Constraint temporal databases
      e. Cognitive/Philosophical temporal reasoning
         i. Usually these groups of people work with syntax and semantics
         ii. high level theories which are not necessarily implemented in computer science
   f. In AI, there is a conference devoted to temporal reasoning TIME/ICTL

4. How does TCSP differ from temporally grounded planning?
   a. In planning, you are thinking about sequences of events, you don’t consider time
   b. Planning is itself very expensive, beyond NP-complete, it is PSPACE
   c. In temporally grounded planning: you include considerations about time duration of actions and states.
   d. Typically, people in planning restrict themselves to using STP

5. Is there no concept of simultaneous actions in planning?
   a. Yes, remember Partial ordered planning (POP) - AI class notes
   b. Also, a graph plan produces actions are in parallel.
Robert’s Question and Comments

1. Provided URL for Allen’s transitivity table
   a. page 337 of the paper
   b. will be posted in piazza
2. How do we get all this relations
   a. no proof of relations in Allen’s paper which is very old
3. Do people in operational research work on temporal networks?
   a. Operation research is a field in mathematics with applications in scheduling, resource allocation, decision and optimization problems. It was started in 1939 during World War II for the optimal placement of submarines to protect boats during the war.
   b. To the best of my knowledge, people in operational research do not develop temporal representations and formalisms. Representation of time is not made *explicit*.
   c. People in operational research may use algorithms developed by mathematicians like Floyd-Warshall.

Tony’s Question and Comments

1. Why is point algebra limited to binary constraints?
   a. This is definition of point algebra
   b. Daniel also mentioned before of other algebra representation which looks beyond binary constraints

Zhang’s Question and Comments

1. Question about Inverse of equal being not equal?
   a. Do not confuse the inverse of a *relation* and the inverse of a Boolean proposition.
   b.
2. Can you combine qualitative and quantitative temporal networks?
   a. Yes. You can have hybrid networks.
   b. Sometimes you can transform quantitative into qualitative but it may be difficult to do the reverse. Anyway, you may lose information.
   c. We use channeling constraint to map qualitative and quantitative constraints. Then you do as much propagation and reasoning as you can in each representation, then channel the inferences from one presentation to the next. Keep repeating until you have reached some desired result.
   d. Robert answered Zhang’s question and he found a summary paper which will be posted by Robert as a comment in his piazza post.

FIKAYO’s Question and Comments

1. Demonstrate an example to determine if a distance graph has negative cycles if it is consistent?
a. No. *Negative cycle means the STP is inconsistent.* Example shown in the next class. Just draw a triangle and have two edges labeled with intervals whose composition does not intersect with the labeling of the third edge.

2. Also if an STP is not consistent, can it be made consistent?
   a. Yes it can be made consistent relaxing some of the constraints. But it can be hard to determine which constraints to relax and by how much.

3. Is there a situation in which ULT becomes a bad option for solving the fragmentation problem?
   a. Left it for future lecture by instructor.
   b. *Proposal by Ali:* I think one such situation maybe when the total number of intervals (n) in the original TCSP instance is very large. ULT algorithm repeats until there is an inconsistency found or at most n times.

**Istiaque’s Question and Comments**

1. In TCSPs, time point $X_0$ represents “beginning of the world”. In Rina Dechter book section 12.4, the authors use $X_0=0$. My question is that depending on the TCSP instance, can we set $X_0$ to some other number and adjust all the constraints $T_0$ accordingly?
   a. We don’t care much about variable values. Once we have a problem that is consistent, the constraints are minimal. Then whichever value you choose for any time point, you can find consistent values for all the other time points.

2. Should we use $\Delta AC$ all the time as preprocessor in TCSPs, or is it sometimes excessive to use $\Delta AC$?
   a. $\Delta AC$ is very cheap and effective
   b. Once you apply $\Delta AC$, the subset usually stays consistent
   c. During search, you can use $\Delta AC$ every time but we have not tested its effectiveness. May be the basis of a full-lookahead strategy for the TCSP.

**Daniel’s Question and Comments**

1. Any solution of $T$ is also a solution of one of its STPs and vice versa. Does it mean there is only solution to STP?
   a. Each STP once you have made it as tight as possible and assuming the intervals are discrete, then STP has many solutions in it, which can be generated backtrack free.

**Robert’s Question and Comments**

1. A typo in the book Figure 12.8 page 347 of the book.
2. How do temporal databases operate?
   a. Depending on what the temporal database provide as services, then you have to provide proof theory (i.e., an algorithm) to allow derivation. Not all temporal databases provide the same services. Some temporal database use Floyd-Warshall and some use search.