Recitation 9

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- (3 min max) Questions about last week’s quiz?
- Questions about lecture / homework so far?
- Given the Poset: \( \{\{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}, \subseteq \)
  1. Find the maximal elements: \( \{1, 2\}, \{1, 3, 4\}, \{2, 3, 4\} \) (because these are not subsets of any other sets in the relation right?)
  2. Find the minimal elements: \( \{1\}, \{2\}, \{4\} \) (again, there are no subsets of these sets in the relation)
  3. Is there a greatest element?: No
  4. Is there a least element?: No
  5. Find all of the upper bounds of \( \{\{2\}, \{4\}\} \): \( \{\{2, 4\}, \{2, 3, 4\}\} \)
  6. Find the least upper bound of \( \{\{2\}, \{4\}\} \): \( \{2, 4\} \)
  7. Find all lower bounds of \( \{\{1, 3, 4\}, \{2, 3, 4\}\} \): \( \{\{3, 4\}, \{4\}\} \)
  8. Find the greatest lower bound of \( \{\{1, 3, 4\}, \{2, 3, 4\}\} \): \( \{3, 4\} \)
- Induction: Example using triominoes for \( 2^n \times 2^n \) checkerboard missing one corner, see page 326.
- Now, problem 5.1.5, using induction prove: \( 1^2 + 3^2 + ... + (2n + 1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3} \) for nonnegative \( n \).
  1. Here we can see the base case is 0 (we want \( n \) to be nonnegative and an integer, note not the same as positive), what is \( P(0) \)? \( P(0) = 1 = \frac{(1)(1)(3)}{3} \)
  2. Show that \( P(0) \) is true: \( 1 = 1 \cdot 1 \cdot \frac{1+3}{3} = 1 \). Therefore \( P(0) \) is true.
  3. What is the inductive hypothesis? \( P(k) \) is true, that is \( 1^2 + 3^2 + ... + (2k + 1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3} \).
  4. We want to prove: \( P(k + 1) \) that is \( 1^2 + 3^2 + ... + (2k + 3)^2 = \frac{(k+2)(2k+3)(2k+5)}{3} \)
5. So we have that $P(k)$ is true, $P(k+1)$ is really just $1^2 + 3^2 + ... + (2k+1)^2 + (2k+3)^2$. By our inductive hypothesis, we have that this is really $\frac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2$. This equals $(2k+3) \cdot (\frac{(k+1)(2k+1)+6k+9}{3}) = (2k+3) \cdot (\frac{2k^3+9k+10}{3}) = \frac{(k+2)(2k+3)(2k+5)}{3}$. This is what we were trying to prove, therefore, by induction, $1^2 + 3^2 + ... + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ for nonnegative $n$.

- Now, prove the following: $3 \mid 2^{2n} - 1$ for $n \geq 1$.

1. Base case is $n = 1$. So $P(1) = 3 \mid 2^{2(1)} - 1$.
2. Well clearly $2^{2(1)} - 1 = 4 - 1 = 3$. And it is obvious that $3 \mid 3$, so the base case is proven.
3. What is our inductive hypothesis? $P(k)$ so $3 \mid 2^{2k} - 1$.
4. We want to prove that $3 \mid 2^{2k+2} - 1$.
5. Well, $P(k + 1) = 3 \mid 2^{2k+2} - 1$. We also can see $2^{2k+2} - 1 = 4 \cdot 2^k - 1 = 4 \cdot (2^k - 1 + 1) - 1$.
6. But we know, by our inductive hypothesis that this equals $4 \cdot (3t + 1) - 1 = 12t + 4 - 1 = 12t + 3 = 3(4t + 1)$.
7. Clearly this is divisible by 3, therefore $P(k + 1) = 3 \mid 2^{2k+2} - 1$ is true.