Recitation 9

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- (3 min max) Questions about last week's quiz?
- Questions about lecture / homework so far?
- Given the Poset: $(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\},\subseteq)$
  1. Find the maximal elements:$\{1,2\},\{1,3,4\},\{2,3,4\}$ (because these are not subsets of any other sets in the relation right?)
  2. Find the minimal elements: $\{1\},\{2\},\{4\}$ (again, there are no subsets of these sets in the relation)
  3. Is there a greatest element?: No
  4. Is there a least element?: No
  5. Find all of the upper bounds of $\{\{2\},\{4\}\}$: $\{\{2,4\},\{2,3,4\}\}$
  6. Find the least upper bound of $\{\{2\},\{4\}\}$: $\{2,4\}$
  7. Find all lower bounds of $\{\{1,3,4\},\{2,3,4\}$: $\{\{3,4\},\{4\}\}$
  8. Find the greatest lower bound of $\{\{1,3,4\},\{2,3,4\}\}$: {3,4}
- Induction: Example using triominoes for $2^n \times 2^n$ checkerboard missing one corner, see page 326.
- Now, problem 5.1.5, using induction prove: $1^2 + 3^2 + ... + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ for nonnegative n.
  1. Here we can see the base case is 0 (we want n to be nonnegative and an integer, note not the same as positive), what is $P(0)$? $P(0) = 1 = \frac{(1)(1)(3)}{3}$
  2. Show that P(0) is true:$1 = 1.\frac{1*1*3}{3} = 1$. Therefore P(0) is true.
  3. What is the inductive hypothesis? $P(k)$ is true, that is $1^2 + 3^2 + ... + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$.
  4. We want to prove: $P(k+1)$ that is $1^2 + 3^2 + ... + (2k+3)^2 = \frac{(k+2)(2k+3)(2k+5)}{3}$

5. So we have that $P(k)$ is true, $P(k+1)$ is really just $1^2+3^2+...+(2k+1)^2+(2k+3)^2$. By our inductive hypothesis, we have that this is really $\frac{(k+1)(2k+1)(2k+3)}{3}+(2k+3)^2$. This equals $(2k+3)*(\frac{(k+1)(2k+1)+6k+9}{3}) = (2k+3)*(\frac{2k^2+9k+10}{3}) = \frac{(k+2)(2k+3)(2k+5)}{3}$...this is what we were trying to prove, therefore, by induction, $1^2+3^2+...+(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ for nonnegative n.

- Now, prove the following: $3 \mid 2^{2n}-1$ for $n \geq 1$.

1. Base case is $n=1$. So $P(1) = 3 \mid 2^{2(1)}-1$.
2. Well clearly $2^{2(1)}-1 = 4-1 = 3$. And it is obvious that $3 \mid 3$, so the base case is proven.
3. What is our inductive hypothesis? $P(k)$ so $3 \mid 2^{2k}-1$.
4. We want to prove that $3 \mid 2^{2k+2}-1$.
5. Well, $P(k+1) = 3 \mid 2^{2k+2}-1$. We also can see $2^{2k+2}-1 = 4*2^k-1 = 4*(2^k-1+1)-1$.
6. But we know, by our inductive hypothesis that this equals $4*(3t+1)-1 = 12t+4-1 = 12t+3 = 3(4t+1)$.
7. Clearly this is divisible by 3, therefore $P(k+1) = 3 \mid 2^{2k+2}-1$ is true.