Recitation 6

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- (3 min max) Questions about the exam, or material covered on the exam.
- Questions about lecture / homework so far?
- So let's look again at some set exercises, here is number 2.2:31:
  - Show that for A and B, subsets of some universal set U, $A \subseteq B$ iff $\bar{B} \subseteq \bar{A}$
  - First $A \subseteq B \equiv \{x \mid x \in A \rightarrow x \in B\}$
  - $\equiv \{x \mid x \notin A \vee x \in B\} \equiv \{x \mid x \in B \vee x \notin A\}$
  - $\equiv \{x \mid x \notin B \rightarrow x \notin A\}$
  - But this is equal to $\bar{B} \subseteq \bar{A}$.
  - So $A \subseteq B$ iff $\bar{B} \subseteq \bar{A}$
- 2.2.37 c:
  - Show $A \oplus U = \bar{A} \equiv \{x \mid (x \in A \wedge x \notin U) \vee (x \notin A \wedge x \in U)$
  - however, if $x \in A$, then $x \in U$, therefore $x \in A \wedge x \notin U$ do not exist.
  - So $A \oplus U = \bar{A} \equiv \{x \mid x \notin A \wedge x \in U\}$
  - But this is the definition of $\bar{A}$ therefore:
  - $A \oplus U = \bar{A}$
- Suppose that $A \cup B = \emptyset$, what can you conclude? (Prove formally)

  Answer: we conclude that $(A = \emptyset) \wedge (B = \emptyset)$.

  The proof is by contradiction. Assume $\neg[(A = \emptyset) \wedge (B = \emptyset)] \Rightarrow (A \neq \emptyset)$ or $(B \neq \emptyset)$ $\Rightarrow$ there is at least an element $a \in A$ or at least an element $b \in B$. Without loss of generality, assume it is $a \in A$, then $a \in A \cup B$, but $A \cup B = \emptyset$, contradiction! Therefore, $A = \emptyset$ and $B = \emptyset$.

  Remember that WLOG is the proof by cases. What we did above is a condensed version of the following two cases:

  1. A is not empty and B is empty $\Rightarrow \exists a \in A$ etc.
  2. A is empty and B is not empty: same as previous case but inverting A and B.

Because the two cases are so similar, we can condense them into one and add the statement WLOG.

- Now let's look at functions, say we have the following function: $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \lfloor \frac{x}{2} \rfloor$
  - First what does the graph of this function look like?
  - is f 1-1 (injective)? No, for example both 1 and 1.1 are assigned 0.
  - Is f **onto** $\mathbb{R}$? No, the floor function only maps to integers, so only integers would be mapped to.
- Let $A = \{1, 2, 3, 4\}$, let$B = \{a, b, c\}$, and let $C = \{2, 7, 10\}$
  - $g : A \to B = \{(1, b), (2, a), (3, a), (4, b)\}$ and $f : B \to C = \{(a, 10), (b, 7), (c, 2)\}$
  - Find $f \circ g$: $\{(1, 7), (2, 10), (3, 10), (4, 7)\}$
  - Findf^{-1}:$\{(10, a), (7, b), (2, c)\}$
  - Is g^{-1} a function? No, because it would map a to multiple values, but functions are defined, such that each element is mapped to *exactly one* element in the codomain.
  - Find $f \circ f^{-1}$: $\{(10, 10), (7, 7), (2, 2)\}$
- Prove or disprove: for all $x, y \in \mathbb{R}$, $\lfloor x \times y \rfloor \leq \lfloor x \rfloor \times \lfloor y \rfloor$
  - let $x = 3.5$, and $y = 1.5$. $\lfloor 3.5 \times 1.5 \rfloor = 5$, but $\lfloor 3.5 \rfloor \times \lfloor 1.5 \rfloor = 4$
  - $5 \neq 4$, therefore this is not true.
- Prove or disprove for all $x, y \in \mathbb{R}$, $\lceil x \times y \rceil \leq \lceil x \rceil \times \lceil y \rceil$
  - Here the same example works $\lceil 3.5 \times 1.5 \rceil = 5$, but $\lceil 3.5 \rceil \times \lceil 1.5 \rceil = 6$
- Show that the function $f(x) = |x|$ from the set of real numbers to the set of nonnegative real numbers is not invertible, but if the domain is restricted to the set of nonnegative real numbers, the resulting function is invertible.

  For a function to be invertible, it needs to be: bijective.

  Therefore, we need to check if this is one-to-one (injective) and onto (surjective).

  1. Injective: No, $f(x_1) \neq f(x_2) \Rightarrow |x_1| \neq |x_2| \Rightarrow \pm x_1 \neq \pm x_2$

     Now, if the domain is restricted to the set of nonnegative real numbers. Is $f(x)$ injective?

     $f(x_1) = f(x_2) \Rightarrow |x_1| = |x_2| \Rightarrow x_1 = x_2$. Therefore, on the restricted domain $f(x)$ is injective.

2. Surjective: For some element $b \in \text{rng}(f) \Leftrightarrow b = |a|$, with $a \in \mathcal{R} \Leftrightarrow b$ is positive.

   Thus, the range is the set of all nonnegative real numbers. Because the range and the codomain are the same, we can conclude that f is surjective.

3. Bijective: No, because it is not injective.

   Though, on the restricted domain, it is bijective because it is both injective and surjective.

4. Invertible: Again, only on the restricted domain.

- Now a quick review of membership, determine whether these statements are true or false:

  1. $\{a, b\} \subseteq \{\{a, b\}\}$

     False, because neither a nor b is an element in $\{\{a, b\}\}$.

  2. $\{a, b\} \in \{\{a, b\}\}$

     True, because there the element $\{a, b\}$ is in $\{\{a, b\}\}$

  3. $\{a, b, c\} \subset \{a, b, c\}$

     False, because the sets are equal, and the statement is wondering if it is a strict subset.

  4. $\{a, b, c\} \subseteq \{a, b, c\}$

     True, because the sets are equal.

  5. $\{\} \subseteq \{a, b, c\}$

     True, because the empty set $\emptyset = \{\}$ is a subset of all sets.

  6. $\emptyset \in \{a, b, c\}$

     False, because the element $\emptyset$ is not in the set $\{a, b, c\}$.

  7. $\{a\} \subset \{a, a\}$

     False, the set $\{a, a\}$ is really $\{a\}$ because, in a set, elements are *not* repeated. Therefore, $\{a\} \subset \{a, a\}$ is *false* because $\{a\} \not\subset \{a\}$ (Note that $\{a\} \subseteq \{a\}$ though).