

# Recitation 5

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- (3 min max) Go over quiz from last week
- (3 min max) Go over homework from last week.
- Questions about lecture / homework so far?
- To start, we'll look at some proofs, here we have 1.7.19:
  - Let  $P(n)$  be the proposition: *If  $a$  and  $b$  are positive real numbers, then  $(a + b)^n \geq a^n + b^n$ .*
  - Prove that  $P(1)$  is true.
    1. First let  $a$  and  $b$  be positive real numbers
    2.  $P(1) = (a + b)^1 \geq a^1 + b^1$ .
    3. So  $P(1) = a + b \geq a + b$ .
    4. Well this is clearly true, since  $a + b = a + b$ .
  - What kind of proof did we use? **direct proof**.
- Now a more difficult proof 1.7.29:
  - Prove or disprove: Given  $m, n \in \mathbb{Z}$ , and  $mn = 1$ , either  $m = 1$  and  $n = 1$ , or  $m = -1$  and  $n = -1$ .
    1. Case 1: Suppose  $|m| > 1$ . Regardless of what  $n$  is,  $mn = 1$ , so  $n = \frac{1}{|m|}$ . But we said that both  $m$  and  $n$  are integers. If  $|m| > 1$ , then  $n$  here cannot be an integer. So, contradiction.
    2. Case 2: Similar to case 1, but suppose  $|n| > 1$ . Completely symmetric, works out the same.
    3. Case 3: Suppose  $m = 1$  and  $n = -1$ . Clearly then  $mn = -1$ . So, contradiction.
    4. Case 4: Suppose  $m = -1$  and  $n = 1$ . Again clearly  $mn = -1$ . So, contradiction.
    5. Case 5: Suppose  $m = 1$  and  $n = 1$ . Then  $mn = 1$ .
    6. Case 6: Suppose  $m = -1$  and  $n = -1$ . Then  $mn = 1$ .

- What techniques did we use? **cases and contradiction**
- 1.8:Example 10: Show that there is a positive integer that can be written as the sum of cubes of positive integers, in two different ways:
  - Here we're going to use a *Constructive Existence* proof.
  - $1729 = 10^3 + 9^3 = 12^3 + 1^3$ .
  - So, 1729 can be written of the sum of cubes of positive integers in two different ways.
  - Therefore there is a positive integer that can be written as the sum of cubes of positive integers, in two different ways.
  - These types of proof are basically finding an example.
  - Unfortunately, other than using your intuition, the only way to go about it is brute force.
- 1.8:Example 20: Can we tile a standard chess board with opposite corners removed using dominos (e.g. upper left, lower right removed)?
  - Suppose we can.
  - First, note that a standard chessboard can be covered by dominos, each having one white and one black square.
  - We know that the standard chess board has 64 square. Removing 2 gives  $64-2 = 62$  squares.
  - Tiling with dominos uses  $62/2 = 31$  dominos.
  - These dominos each have one white and one black square (like chessboards have white and black).
  - So using the tiles we have 31 white and 31 black squares.
  - However, when we remove two opposite corner squares, either 32 of the remaining squares are white and 30 are black, or 32 are black and 30 are white.
  - But we said we had 31 of each color...contradiction
  - Therefore we cannot tile a standard chessboard with two opposite corners removed, with dominos.

- Now moving on to sets, 2.1.11: Determine whether true or false:

a  $x \in \{x\}$

\* true

b  $\{x\} \subseteq \{x\}$

\* true

c  $\{x\} \in \{x\}$

\* false

d  $\{x\} \in \{\{x\}\}$

\* true

e  $\emptyset \subseteq \{x\}$

\* true

f  $\emptyset \in \{x\}$

\* false

- 2.1.27(a) Let  $A = \{a, b, c, d\}$ , and  $B = \{y, z\}$ .

a  $A \times B$ ?

1.  $|A \times B|$ ? As in, how big will it be?  $4 \times 2 = 8$

2.  $\{(a, y), (b, y), (c, y), (d, y), (a, z), (b, z), (c, z), (d, z)\}$

– How about  $\mathbb{Z} \times (\mathbb{Z}^+ \setminus 0)$

1. What is its *cardinality*?  $\infty$

2. What are some of its elements?  $(1,2), (3,4), (-1,5)$ .

3. Is  $(2, 0) \in \mathbb{Z} \times (\mathbb{Z}^+ \setminus 0)$ ? No

4. Does this set remind you of anything? Could be used to represent the rational numbers,  $\mathbb{Q}$ .

- Computer the power set of the following set  $S = \{a, b, \{c\}, \emptyset\}$

– First, how many elements are in  $\mathcal{P}(S)$ ?  $2^4 = 16$ .

–  $\{\emptyset, \{\emptyset\}, \{\{c\}\}, \{a\}, \{b\}, \{a, b\}, \{a, \{c\}\}, \{b, \{c\}\}, \{a, \emptyset\}, \{b, \emptyset\}, \{\{c\}, \emptyset\}, \{a, b, \{c\}\}, \{b, \{c\}, \emptyset\}, \{a, \{c\}, \emptyset\}, \{a, b, \{c\}, \emptyset\}$

- (Last 15 minutes) Give quiz