Computer Science & Engineering 235 – Discrete Mathematics

Logical Equivalences, Implications, Inferences, and Set Identities

1.	$\neg(\neg p) \iff p$	Double Negation
2.a.	$(p \vee q) \iff (q \vee p)$	Commutative Laws
2.b.	$(p \wedge q) \iff (q \wedge p)$	
2.c.	$(p \leftrightarrow q) \iff (q \leftrightarrow p)$	
3.a.	$[(p \vee q) \vee r] \iff [p \vee (q \vee r)]$	Associative Laws
3.b.	$[(p \wedge q) \wedge r] \iff [p \wedge (q \wedge r)]$	
4.a.	$[p \vee (q \wedge r)] \iff [(p \vee q) \wedge (p \vee r)]$	Distributive Laws
4.b.	$[p \wedge (q \vee r)] \iff [(p \wedge q) \vee (p \wedge r)]$	
5.a.	$(p \vee p) \iff p$	Idempotent Laws
5.b.	$(p \wedge p) \iff p$	
6.a.	$(p \vee 0) \iff p$	Identity Laws
6.b.	$(p \wedge 1) \iff p$	
7.a.	$(p \vee 1) \iff 1$	Domination Law
7.b.	$(p \wedge 0) \iff 0$	
8.a.	$\neg(p \vee q) \iff (\neg p \wedge \neg q)$	DeMorgan's Laws
8.b.	$\neg(p \wedge q) \iff (\neg p \vee \neg q)$	
8.c.	$(p \vee q) \iff \neg(\neg p \wedge \neg q)$	
8.d.	$(p \wedge q) \iff \neg(\neg p \vee \neg q)$	
9.	$(p \to q) \iff (\neg q \to \neg p)$	Contrapositive
10.a.	$(p \to q) \iff (\neg p \vee q)$	Implication
10.b.	$(p \to q) \iff \neg(p \wedge \neg q)$	
11.a.	$(p \vee q) \iff (\neg p \to q)$	
11.b.	$(p \wedge q) \iff \neg(p \to \neg q)$	
12.a.	$[(p \to r) \wedge (q \to r)] \iff [(p \vee q) \to r]$	
12.b.	$[(p \to q) \wedge (p \to r)] \iff [p \to (q \wedge r)]$	
13.	$(p \leftrightarrow q) \iff [(p \to q) \wedge (q \to p)]$	Equivalence
14.	$[(p \wedge q) \to r] \iff [p \to (q \to r)]$	Exportation Law
15.	$(p \to q) \iff [(p \wedge \neg q) \to c]$	Reductio ad Absurdum
16.	$(p \oplus q) \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Exclusive Or
17.a.	$p \vee (p \wedge q) \equiv p$	Absorption Laws
17.b.	$p \wedge (p \vee q) \equiv p$	
18.a.	$(p \vee \neg p) \iff 1$	Negation Laws
18.b.	$(p \wedge \neg p) \iff 0$	

Table 1: Logical Equivalences

1.	$p \Rightarrow (p \vee q)$	Addition
2.	$(p \wedge q) \Rightarrow p$	Simplification
3.	$(p \rightarrow c) \Rightarrow \neg p$	Absurdity
4.	$[p \wedge (p \rightarrow q)] \Rightarrow q$	Modus Ponens
5.	$[(p \rightarrow q) \wedge \neg q] \Rightarrow \neg p$	Modus Tollens
6.	$[(p \vee q) \wedge \neg p] \Rightarrow q$	Disjunctive Syllogism
7.	$p \Rightarrow [q \rightarrow (p \wedge q)]$	
8.a.	$[p \leftrightarrow q) \wedge (q \leftrightarrow r)] \Rightarrow (p \leftrightarrow r)$	Transitivity
8.b.	$[p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$	
9.a.	$(p \rightarrow q) \Rightarrow [(p \vee r) \rightarrow (q \vee r)]$	
9.b.	$(p \rightarrow q) \Rightarrow [(p \wedge r) \rightarrow (q \wedge r)]$	
9.c.	$(p \rightarrow q) \Rightarrow [(p \rightarrow r) \rightarrow (q \rightarrow r)]$	
10.a.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \Rightarrow [(p \vee r) \rightarrow (q \vee s)]$	Constructive Dilemmas
10.b.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \Rightarrow [(p \wedge r) \rightarrow (q \wedge s)]$	
11.a.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \Rightarrow [(\neg q \vee \neg s) \rightarrow (\neg p \vee \neg r)]$	Destructive Dilemmas
11.b.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \Rightarrow [(\neg q \wedge \neg s) \rightarrow (\neg p \wedge \neg r)]$	

Table 2: Logical Implications

$A \cup \emptyset = A$	Identity Laws
$A \cap U = A$	
$A \cup U = U$	Domination Laws
$A \cap \emptyset = \emptyset$	
$A \cup A = A$	Idempotent Laws
$A \cap A = A$	
$\overline{(\overline{A})} = A$	Complementation Law
$A \cup B = B \cup A$	Commutative Laws
$A \cap B = B \cap A$	
$A \cup (B \cup C) = (A \cup B) \cup C$	Associative Laws
$A \cap (B \cap C) = (A \cap B) \cap C$	
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive Laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	
$\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's Laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$	
$A \cup (A \cap B) = A$	Absorption Laws
$A \cap (A \cup B) = A$	
$A \cup \overline{A} = U$	Complement Laws
$A \cap \overline{A} = \emptyset$	

Table 3: Set Identities