Algorithms: An Introduction

'Algorithm' is a distortion of Al-Khawarizmi, a Persian mathematician

Section 3.1 of Rosen

Spring 2013

CSCE 235 Introduction to Discrete Structures

Course web-page: cse.unl.edu/~cse235

Questions: Piazza

Outline

- Introduction & definition
- Algorithms categories & types
- Pseudo-code
- Designing an algorithm
  - Example: MAX
- Greedy Algorithms
  - CHANGE

Computer Science is About Problem Solving

- A Problem is specified by
  1. **The givens** (a formulation)
     - A set of objects
     - Relations between them
  2. **The query**
     - The information one wants to extract from the formulation, the question to answer

Real World	↔	Computing World
Objects	represented by…	data Structures, ADTs, Classes
Relations	implemented with…	relations & functions (e.g., predicates)
Actions	Implemented with…	algorithms: a sequence of instructions

- An algorithm is a method or procedure that solves instances of a problem

Algorithms: Formal Definition

- **Definition**: An algorithm is a sequence of unambiguous instructions for solving a problem.
- Properties of an algorithm
  - **Finite**: the algorithm must eventually terminate
  - **Complete**: Always give a solution when one exists
  - **Correct (sound)**: Always give a correct solution
- For an algorithm to be an acceptable solution to a problem, it must also be effective. That is, it must give a solution in a 'reasonable' amount of time
- Efficient= runs in polynomial time. Thus, **effective≠ efficient**
- There can be many algorithms to solve the same problem

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Algorithms: General Techniques

- There are many broad categories of algorithms
  - Deterministic versus Randomized (e.g., Monte-Carlo)
  - Exact versus Approximation
  - Sequential/serial versus Parallel, etc.
- Some general styles of algorithms include
  - Brute force (enumerative techniques, exhaustive search)
  - Divide & Conquer
  - Transform & Conquer (reformulation)
  - Greedy Techniques

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Good Pseudo-Code: Example

INTERSECTION

Input: Two finite sets A, B

Output: A finite set C such that $C = A \cap B$

```
C ← ∅
If |A|>|B|
  Then SWAP(A,B)
End
For every x ∈ A Do
  If x ∈ B
    Then C ← C ∪ {x}                UNION(C,{x})
  End
End
Return C
```

Algorithms: Pseudo-Code

- Algorithms are usually presented using <u>pseudo-code</u>
- Bad pseudo-code
  - gives too many details or
  - is too implementation specific (i.e., actual C++ or Java code or giving every step of a sub-process such as set union)
- Good pseudo-code
  - Is a balance between clarity and detail
  - Abstracts the algorithm
  - Makes good use of mathematical notation
  - Is easy to read and
  - Facilitates implementation (reproducible, does not hide away important information)

Writing Pseudo-Code: Advice

- Input/output must properly defined
- All your variables must be properly initialized, introduced
- Variables are instantiated, assigned using $\leftarrow$
- All `commands' (while, if, repeat, begin, end) bold face \bf

  For *i* ← 1 to *n* **Do**

- All functions in small caps UNION(*s*,*t*) \sc
- All constants in courier: `pi` ← 3.14 \tt
- All variables in italic: *temperature* ← 78 (\it, \em)
- LaTeX: Several algorithm formatting packages exist on WWW

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Designing an Algorithm

- A general approach to designing algorithms is as follows
  - Understanding the problem, **assess its difficulty**
  - Choose an approach (e.g., exact/approximate, deterministic/probabilistic)
  - (Choose appropriate data structures)
  - Choose a strategy
  - Prove
    1. Termination
    2. Completeness
    3. Correctness/soundness
  - Evaluate complexity
  - Implement and test it
  - Compare to other known approach <u>and</u> algorithms

Algorithm Example: Max

- When designing an algorithm, we usually give a formal statement about the problem to solve
- **Problem**
  - **Given**: a set $A=\{a_1,a_2,\ldots,a_n\}$ of integers
  - **Question**: find the index i of the maximum integer a_i
- A straightforward idea is
  - Simply store an initial maximum, say a_1
  - Compare the stored maximum to every other integer in A
  - Update the stored maximum if a new maximum is ever encountered

Pseudo-code of Max

MAX

Input: A finite set $A=\{a_1,a_2,\ldots,a_n\}$ of integers

Output: The largest element in the set

1. *temp* $\leftarrow a_1$
2. **For** i =2 **to** n **Do**
3.   **If** $a_i >$ *temp*
4.   **Then** *temp* $\leftarrow a_i$
5.   **End**
6. **End**
7. **Return** *temp*

Algorithms: Other Examples

- Check Bubble Sort and Insertion Sort in your textbooks
- … which you should have seen ad nauseum in CSE 155 and CSE 156
- And which you will see again in CSE 310
- Let us know if you have any questions

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Greedy Algorithms

- In many problems, we wish to not only find a solution, but to find the best or optimal solution
- A simple technique that works for some optimization problems is called the greedy technique
- As the name suggests, we solve a problem by being greedy:
  - Choose what appears now to be the best choice
  - Choose the most immediate best solution (i.e., think locally)
- Greedy algorithms
  - Work well on some (simple) algorithms
  - Usually they are not guaranteed to produce the best globally optimal solution

Change-Making Problem

- We want to give change to a customer but we want to minimize the number of total coins we give them
- **Problem**
  - **Given**: An integer n an a set of coin denominations $(c_1,c_2,\ldots,c_r)$ with $c_1>c_2>\ldots>c_r$
  - **Query**: Find a set of coins $d_1,d_2,\ldots,d_k$ such that

$$\Sigma_{i=1}^{k} d_i = n \text{ and } k \text{ is minimized}$$

Greedy Algorithm: CHANGE

CHANGE

Input: An integer n and a set of coin denominations $\{c_1, c_2, \ldots, c_r\}$ with $c_1 > c_2 > \ldots > c_r$

Output: A set of coins $d_1, d_2, \ldots, d_k$ such that $\sum_{i=1}^{k} d_i = n$ and k is minimized

1. **For** $i = 1$ **to** r **Do**
2. di $\leftarrow 0$
3. **While** $n \geq c_i$ **Do**
4. $d_i \leftarrow d_i + 1$
5. $n \leftarrow n - c_i$
6. **End**
7. **Return** $\{d_i\}$

CHANGE: Analysis (1)

- Will the algorithm always produce an optimal answer?
- Example
  - Consider a coinage system where c_1=20, c_2=15, c_3=7, c_4=1
  - We want to give 22 'cents' in change
- What is the output of the algorithm?
- Is it optimal?
- It is not optimal because it would give us two c4 and one c1 (3 coins). The optimal change is one c2 and one c3 (2 coins)

CHANGE: Analysis (2)

- What about the US currency system: is the algorithm correct in this case?
- Yes, in fact it is. We can prove it by contradiction.
- For simplicity, let us consider

$$c_1=25,\ c_2=10,\ c_3=5,\ c_4=1$$

Optimality of CHANGE (1)

- Let $C=\{d_1,d_2,\ldots,d_k\}$ be the solution given by the greedy algorithm for some integer n.
- By way of contradiction, assume there is a better solution $C'=\{d_1',d_2',\ldots,d_l'\}$ with $l<k$
- Consider the case of quarters. Say there are q quarters in C and q' in C'.
  1. If q'>q, the greedy algorithm would have used q' by construction. Thus, it is impossible that the greedy uses q<q'.

  2. Since the greedy algorithms uses as many quarters as possible, n=q(25)+r, where r<25. If q'<q, then, n=q'(25)+r' where r'≥25. C' will have to use more smaller coins to make up for the large r'. Thus C' is not the optimal solution.
  3. If q=q', then we continue the argument on the smaller denomination (e.g., dimes). Eventually, we reach a contradiction.
- Thus, C=C' is our optimal solution

Optimality of CHANGE (2)

- But, how about the previous counterexample? Why (and where) does this proof?
- We need the following lemma:

  If n is a positive integer, then n cents in change using quarters, dimes, nickels, and pennies using the fewest coins possible

  - Has at most two dimes
  - Has at most one nickel
  - Has at most four pennies, and
  - Cannot have two dimes and a nickel

  The amount of change in dimes, nickels, and pennies cannot exceed 24 cents

Greedy Algorithm: Another Example

- Check the problem of Scenario I, page 25 in the slides IntroductiontoCSE235.ppt
- We discussed then (remember?) a greedy algorithm for accommodating the maximum number of customers. The algorithm
  - terminates, is complete, sound, and satisfies the maximum number of customers (finds an optimal solution)
  - runs in time linear in the number of customers

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