Title: Adverserial Search

AIMA: Chapter 5 (Sections 5.1, 5.2 and 5.3)

Introduction to Artificial Intelligence

CSCE 476-876, Spring 2012

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# Outline

- Introduction
- Minimax algorithm
- Alpha-beta pruning

## Context

- In an MAS, agents affect each other's welfare
- Environment can be cooperative or competitive
- Competitive environments yield adverserial search problems (games)
- Approaches: mathematical game theory and AI games

## Game theory vs. AI

- AI games: fully observable, deterministic environments, players alternate, utility values are equal (draw) or opposite (winner/loser)
  In vocabulary of game theory: deterministic, turn-taking, two-player, zero-sum games of perfect information
- Games are attractive to AI: states simple to represent, agents restricted to a small number of actions, outcome defined by simple rules
  Not croquet or ice hockey, but typically board games
  Exception: Soccer (Robocup `www.robocup.org/`)

## **Board game playing:** an appealing target of AI research

Board game: Chess (since early AI), Othello, Go, Backgammon, etc.

- Easy to represent
- Fairly small numbers of well-defined actions
- Environment fairly accessible
- Good abstraction of an enemy, w/o real-life (or war) risks :—)

But also: Bridge, ping-pong, etc.

## Characteristics

- 'Unpredictable' opponent: contingency problem
  (interleaves search and execution)
- Not the usual type of 'uncertainty':
  no randomness/no missing information (such as in traffic)
  but, the moves of the opponent expectedly non benign
- Challenges:
  - huge branching factor
  - large solution space
  - Computing optimal solution is infeasible
  - Yet, decisions must be made. Forget A*...

# Discussion

- What are the theoretically best moves?
- Techniques for choosing a good move when time is tight
  $\surd$ Pruning: ignore irrelevant portions of the search space
  $\times$ Evaluation function: approximate the true utility of a state without doing search

## Two-person Games

- 2 player: Min and Max
- Max moves first
- Players alternate until end of game
- Gain awarded to player/penalty give to loser

**Game as a search problem:**

- Initial state: board position & indication whose turn it is
- Successor function: defining legal moves a player can take
  Returns {(move, state)*}
- Terminal test: determining when game is over
  states satisfy the test: terminal states
- Utility function (a.k.a. payoff function): numerical value for outcome e.g., Chess: win=1, loss=-1, draw=0

## Usual search

Max finds a sequence of operators yielding a terminal goal scoring winner according to the utility function

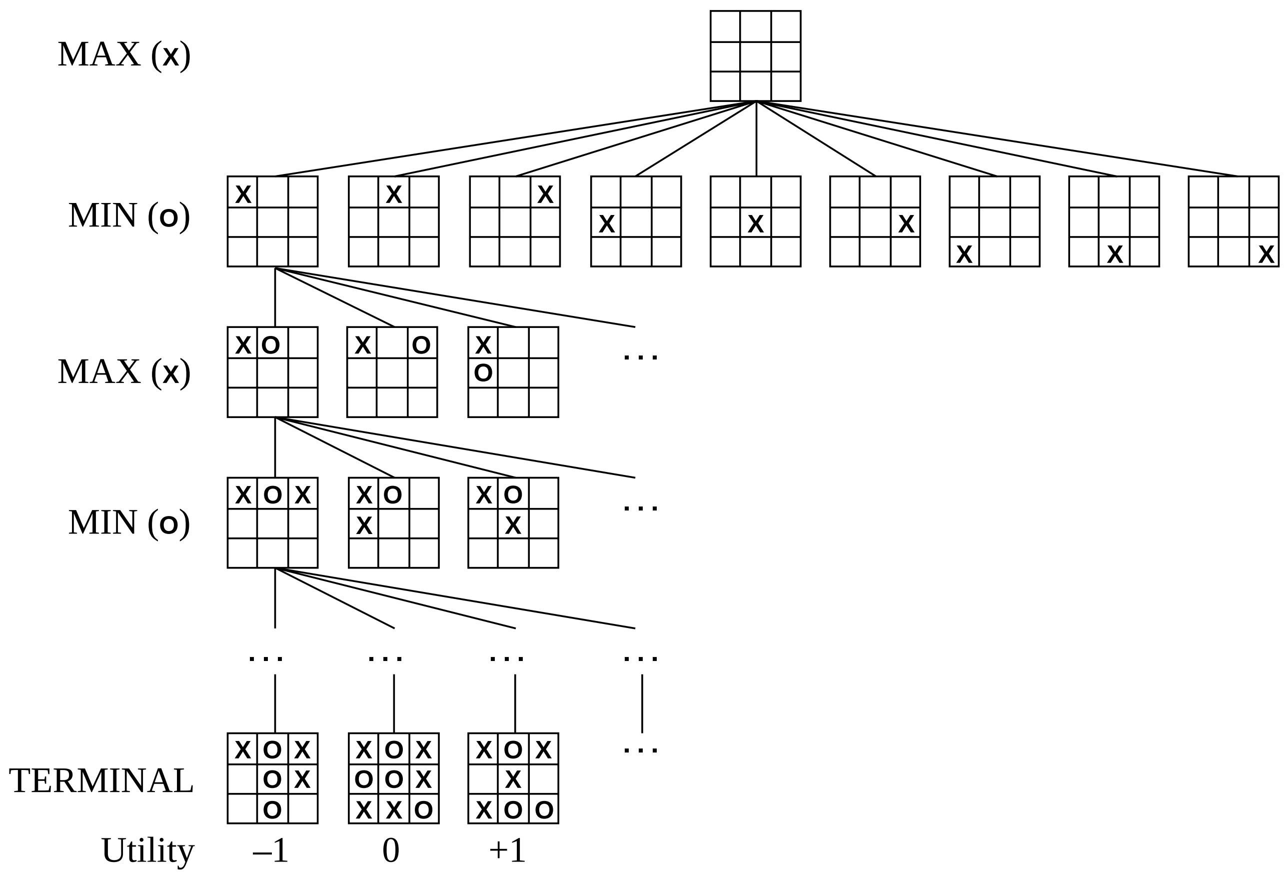
## Game search

- Min actions are significant
  Max must find a strategy to win regardless of what Min does:
  $\longrightarrow$ correct action for Max for each action of Min
- Need to approximate (no time to envisage all possibilities difficulty): a huge state space, an even more huge search space
  *e.g.*, chess: $\begin{cases} 10^{40} \text{ different legal positions} \\ \text{Average branching factor=35, 50 moves/player=} \ 35^{100} \end{cases}$
- Performance in terms of time is very important

# **Example**: Tic-Tac-Toe

Max has 9 alternative moves

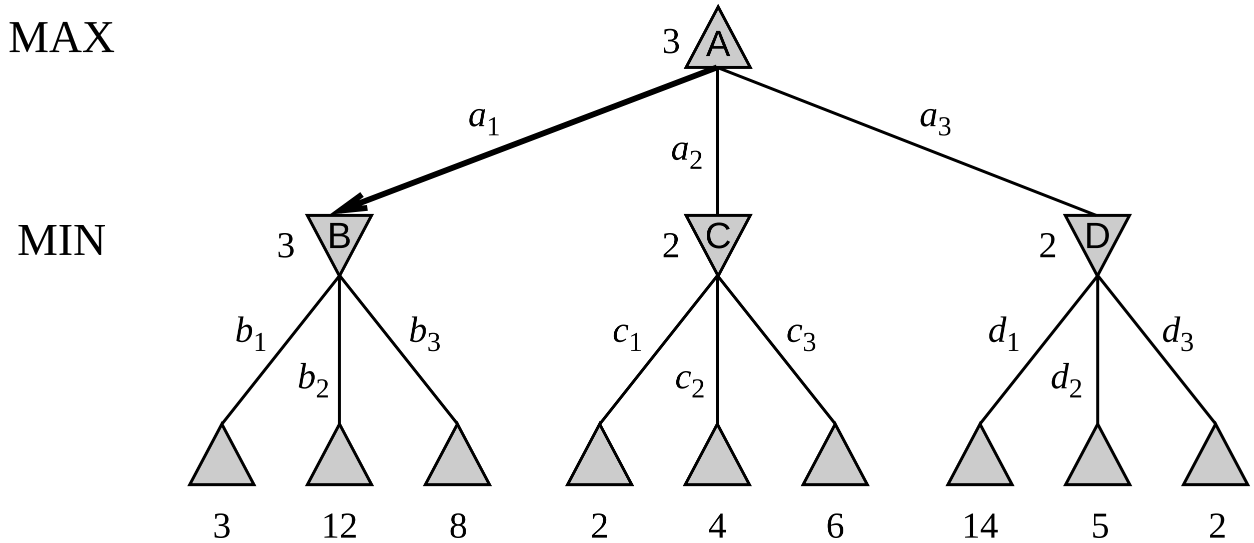
Terminal states' utility: Max wins=1, Max loses = -1, Draw = 0



**Example**: 2-ply game tree

Max's actions: $a_1$, $a_2$, $a_3$

Min's actions: $b_1$, $b_2$, $b_3$




Minimax algorithm determines the optimal strategy for Max

→ decides which is the best move

# Minimax algorithm

- Generate the whole tree, down to the leaves
- Compute utility of each terminal state
- Iteratively, from the leaves up to the root, use utility of nodes at depth $d$ to compute utility of nodes at depth $(d-1)$:

  MIN 'row': minimum of children

  MAX 'row': maximum of children

MINIMAX-VALUE $(n)$

$$\begin{cases} \text{UTILITY}(n) & \text{if } n \text{ is a terminal node} \\ max_{s \in Succ(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a Max node} \\ min_{s \in Succ(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a Min node} \end{cases}$$

## Minimax decision

- MAX's decision: minimax decision maximizes utility under the assumption that the opponent will play perfectly to his/her own advantage
- Minimax decision maximes the worst-case outcome for Max (which otherwise is guaranteed to do better)
- If opponent is sub-optimal, other strategies may reach better outcome better than the minimax decision

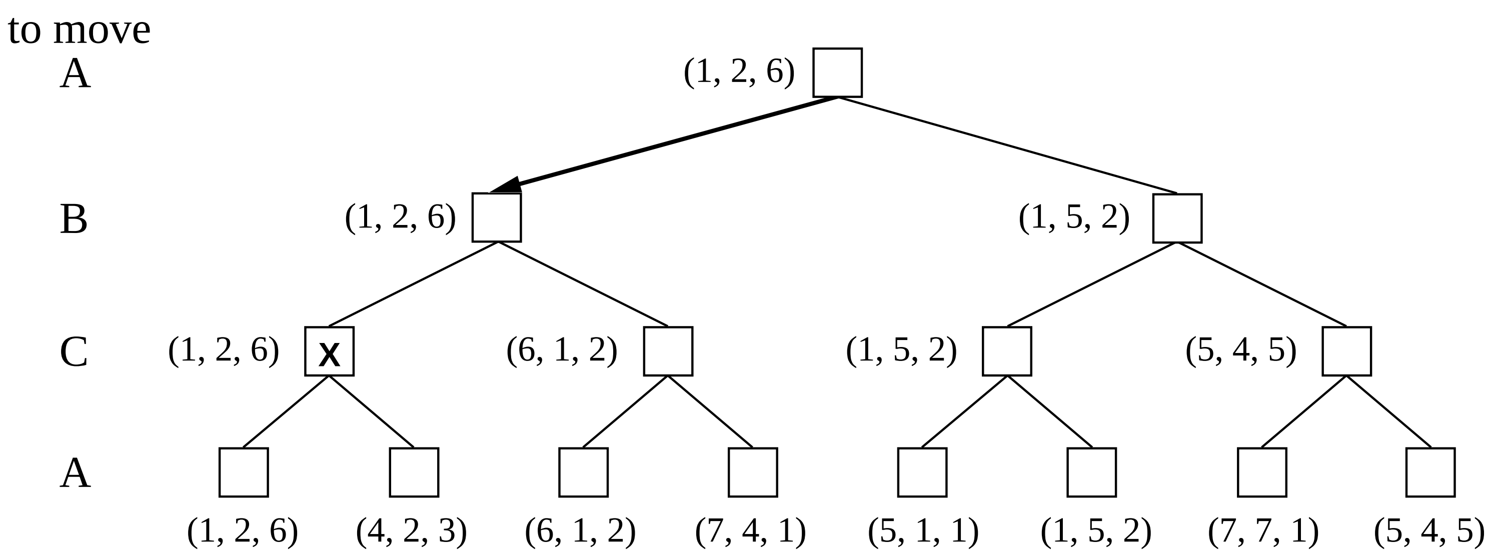
## **Minimax algorithm:** Properties

- $m$ maximum depth
  $b$ legal moves
- Using Depth-first search, space requirement is:
  $O(bm)$: if generating all successors at once
  $O(m)$: if considering successors one at a time
- Time complexity $O(b^m)$
  Real games: time cost totally unacceptable

## Multiple players games

UTILITY($n$) becomes a vector of the size of the number of players

For each node, the vector gives the utility of the state for each player



## **Alliance formation** in multiple players games

How about alliances?

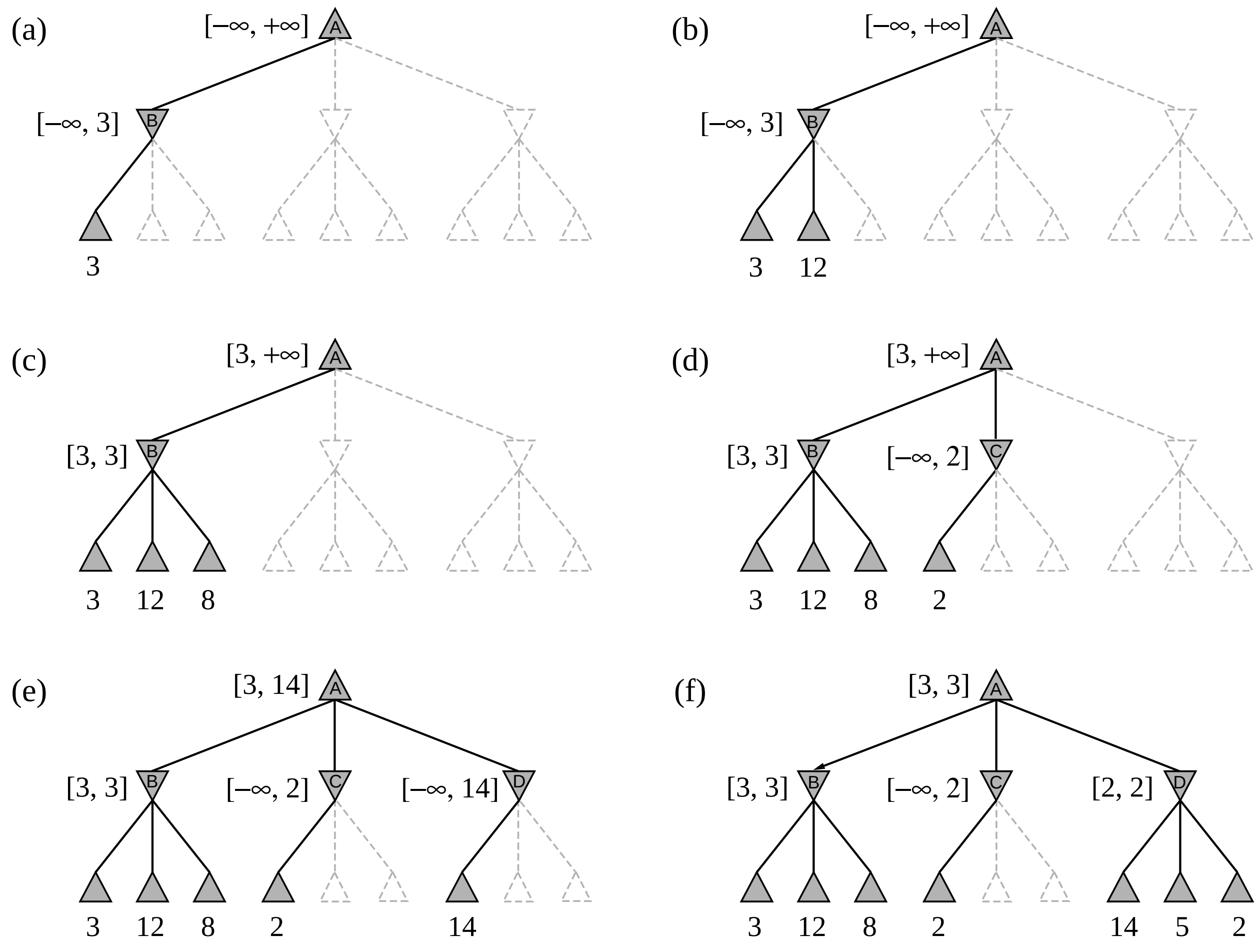
- A and B in weak positions, but C in strong position
  A and B make an alliance to attack C (rather than each other
  → Collaboration emerges from purely selfish behavior!
- Alliances can be done and undone (careful for social stigma!)
- When a two-player game is not zero-sum, players may end up automatically making alliances (for example when the terminal state maximizes utility of both players)

## Alpha-beta pruning

- Minimax requires computing all terminal nodes: unacceptable
- Do we really need to do compute utility of <u>all</u> terminal nodes?
  ... No, says John McCarthy in 1956:

  *It is possible to compute the correct minimax decision without looking at every node in the tree, and yet get the correct decision*
- Use pruning (eliminating useless branches in a tree)

# **Example** of alpha-beta pruning

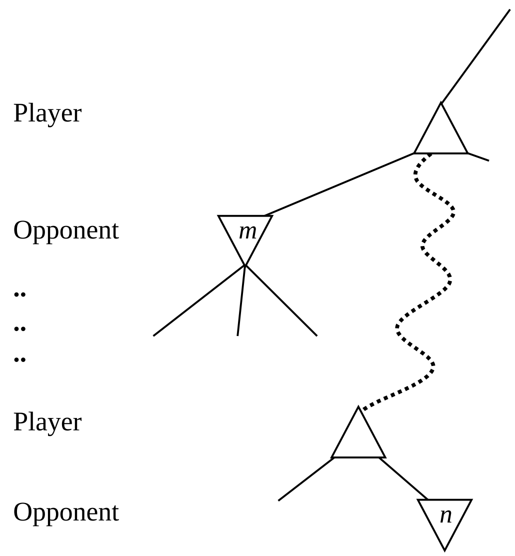



Try 14, 5, 2, 6 below D

# **General principal** of Alpha-beta pruning

If Player has a better choice $m$ at
- — a parent node of $n$
- — any choice point further up

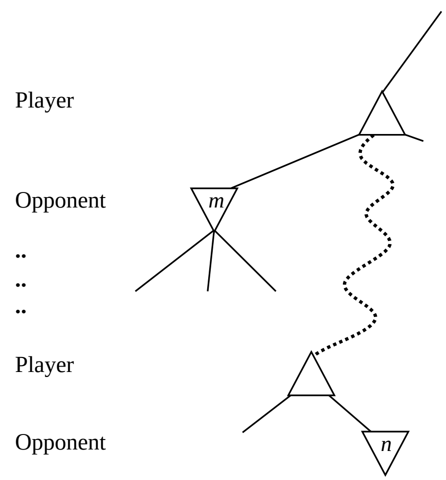
$n$ will never be reached in actual play




Once we have found enough about $n$ (*e.g.*, through one of it descendants), we can prune it (*i.e.*, discard all its remaining descendants)

# **Mechanism** of Alpha-beta pruning

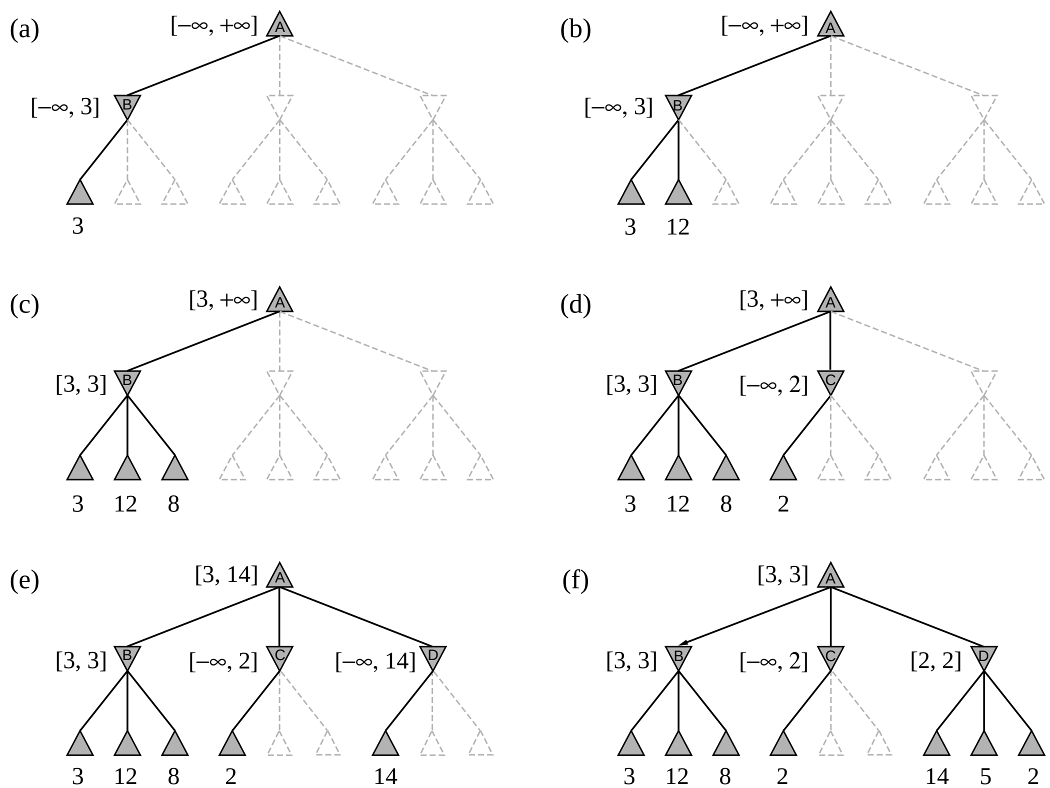
$\alpha$: value of best choice so far for MAX, (maximum)
$\beta$: value of best choice so far for MIN, (minimum)




Alpha-beta search:
- updates the value of $\alpha$, $\beta$ as it goes along
- prunes a subtree as soon as its worse then current $\alpha$ or $\beta$

# Effectiveness of pruning

Effectiveness of pruning depends on the order of new nodes examined



## **Savings** in terms of cost

- Ideal case:
  Alpha-beta examines $O(b^{d/2})$ nodes (vs. Minimax: $O(b^d)$)
  $\rightarrow$ Effective branching factor $\sqrt{b}$ (vs. Minimax: $b$)

- Successors ordered randomly:
  $b > 1000$, asymptotic complexity is $O((b/\log b)^d)$
  $b$ reasonable, asymptotic complexity is $O(b^{3d/4})$

- Practically: Fairly simple heuristics work (fairly) well