Recitation Week 3

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- Questions about Piazza, LaTeX or lecture?
- Questions on the homework?
- (**Skipped in Recitation**) Let's start by looking at section 1.1, problem 15 on page 14 of your Rosen textbook

  1. First, lets figure out what our *terms* are:
     - p : Grizzly bears have been in the area.
     - q : Hiking is safe on the trail.
     - r : Berries are ripe along the trail.
  2. Now why don't we try converting the following sentences into logical connectives, our *clauses*:
     a : "Berries are ripe along the trail, but grizzly bears have not been seen in the area:" $r \wedge \neg p$

     b : "Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail:" $\neg p \wedge q \wedge r$

     c : "If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area:" $r \rightarrow (q \leftrightarrow \neg p)$

     d : "It is not safe to hike on the trail, but grizzly bears have not been seen in the area and berries along the trail are ripe:" $\neg q \wedge \neg p \wedge r$

     e : "For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area:" $(q \rightarrow (\neg r \wedge \neg p)) \wedge \neg((\neg r \wedge \neg p) \rightarrow q)$

     f : "Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail:" $(p \wedge r) \rightarrow \neg q$

- Now let's look at section 1.1, problem 35, part C. Construct the truth table for the following $(p \to q) \vee (\neg p \to q)$

  1. First we break the problem into 3 parts
     - $p \to q$
     - $\neg p \to q$
     - $(p \to q) \vee (\neg p \to q)$
  2. Now we construct a table filling in the p an q values:

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
0	0	-	-	-
0	1	-	-	-
1	0	-	-	-
1	1	-	-	-

  3. Now fill in the rest of the table:

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
0	0	1	0	1
0	1	1	1	1
1	0	0	1	1
1	1	1	1	1

- Now a harder problem, from section 1.2, problem 35.

  1. First decide what the *terms* are:
     - Let b be the butler
     - Let c be the cook
     - Let g be the gardener
     - Let h be the handyman

     We will use 0 to represent that the person is lying, and a 1 to represent that the person is telling the truth.
  2. Next we parse the question to get our *clauses*:
     - "If the butler is telling the truth then so is the cook." $b \to c$
     - "The cook and the gardener cannot both be telling the truth." $\neg(c \wedge g) \equiv \neg c \vee \neg g$
     - "The gardener and the handyman are not both lying." $\neg(\neg g \wedge \neg h) \equiv g \vee h$
     - "If the handyman is telling the truth, then the cook is lying." $h \to \neg c$

3. Now we want to find a *model* for our *sentence*. Remember a *model* is a truth assignment of our *terms* which satisfies all of our *clauses*. Since we are looking for a *model*, we can stop looking at an assignment once we see that a *clause* evaluates to false. A "-" is used in the following table to denote that we stopped before evaluating that *clause*.

b	c	g	h	$b \to c$	$\neg c \vee \neg g$	$g \vee h$	$h \to \neg c$
0	0	0	0	1	1	0	-
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	-
0	1	0	1	1	1	1	0
0	1	1	0	1	0	-	-
0	1	1	1	1	0	-	-
1	0	0	0	0	-	-	-
1	0	0	1	0	-	-	-
1	0	1	0	0	-	-	-
1	0	1	1	0	-	-	-
1	1	0	0	1	1	0	-
1	1	0	1	1	1	1	0
1	1	1	0	1	0	-	-
1	1	1	1	1	0	-	-

4. From this table we can conclude that the butler and the cook are both lying, and that either the gardener, the handyman, or both are telling the truth.

- Now let's look at Section 1.1 problem number 27 (b) on page 15 of the text:

  - "I come to class whenever there is going to be a quiz." Notice that this can be rephrased "If there is going to be a quiz, I come to class." $p \to q$
  - What is the inverse of this sentence, $\neg p \to \neg q$? "If there is not going to be a quiz, then I do not come to class."
  - What is the converse of this sentence, $q \to p$? "If I come to class, then there is a quiz."
  - What is the contrapositive of this sentence, $\neg q \to \neg p$? "If I do not come to class, then there will not be a quiz."
  - Let's compare the truth tables for the inverse, contrapositive, and converse with implication.

p	q	$p \to q$	$\neg p \to \neg q$	$q \to p$	$\neg q \to \neg p$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	1	0
1	1	1	1	1	1

- Now let's look at a different kind of problem, section 1.1 problem 17 (c) on page 14.
  - Let p stand for $1 + 1 = 3$
  - Let q stand for "Dogs can fly."
  - What is $p \to q$? $1 \to 0$ which is false.
- 1.1 number 39, pg 15: Construct the truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
  - First, can you tell immediately how many rows our truth table will have? With n terms, we get 2^n rows.

a	b	c	d	$(p \leftrightarrow q)$	$(r \leftrightarrow s)$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
0	0	0	0	1	1	1
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	1	1
0	1	0	0	0	1	0
0	1	0	1	0	0	1
0	1	1	0	0	0	1
0	1	1	1	0	1	0
1	0	0	0	0	1	0
1	0	0	1	0	0	1
1	0	1	0	0	0	1
1	0	1	1	0	1	0
1	1	0	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	1	1

- Now let's try looking at a problem using logical equivalences. From section 1.3, number 29 on page 35:
  Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology
  - First we will do so by using a truth table, then we will do so using the logical equivalences.
  - Recall that $\wedge$ has higher precedence than $\to$. We begin by constructing the truth table:

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \land (q \to r) \to (p \to r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

- We can see that every truth assignment here evaluates to true, therefore we have that this is a tautology.
- Now let's try proving it using logical equivalences:
  1. $(p \to q) \land (q \to r) \to (p \to r)$ — Given
  2. $\neg((p \to q) \land (q \to r)) \lor (p \to r)$ — Implication
  3. $\neg((\neg p \lor q) \land (\neg q \lor r)) \lor (\neg p \lor r)$ — Implication $\times$ 3
  4. $\neg(\neg p \lor q) \lor \neg(\neg q \lor r) \lor (\neg p \lor r)$ — DeMorgan's Law
  5. $(p \land \neg q) \lor (q \land \neg r) \lor (\neg p \lor r)$ — DeMorgan's Law $\times 2$ and Double Negation
  6. $(p \land \neg q) \lor (q \land \neg r) \lor \neg p \lor r$ — Associative Law
  7. $(p \land \neg q) \lor \neg p \lor (q \land \neg r) \lor r$ — Commutative Law
  8. $((p \lor \neg p) \land (\neg p \lor \neg q)) \lor (q \land \neg r) \lor r$ — Distributive Law
  9. $(1 \land (\neg p \lor \neg q)) \lor (q \land \neg r) \lor r$ — $p \lor \neg p = 1$
  10. $(\neg p \lor \neg q) \lor (q \land \neg r) \lor r$ — Identity
  11. $(\neg p \lor \neg q) \lor ((q \lor r) \land (\neg r \lor r))$ — Distributive Law
  12. $(\neg p \lor \neg q) \lor ((q \lor r) \land 1)$ — $r \lor \neg r = 1$
  13. $(\neg p \lor \neg q) \lor (q \lor r)$ — Identity
  14. $\neg p \lor \neg q \lor q \lor r$ — Associative Law
  15. $\neg p \lor 1 \lor r$ — $q \lor \neg q = 1$
  16. 1 — Identity
- Thus we have a tautology.

* Now how do we show that 2 things are not logically equivalent, here we see 1.3 problem 31:
  Show that $(p \to q) \to r$ is not logically equivalent to $p \to (q \to r)$.
  - Here we can use a counterexample. Suppose $p = 0$, $q = 1$, and $r = 0$. We can see that $(p \to q) \to r$ is false, however $p \to (q \to r)$ is true.

- Another useful procedure is to be able to show that two compound propositions are equal. Well to do that we can simply use the logical equivalences. For example, problem number 27 in section 1.3 on page 35:
  - Show that $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

1. $p \leftrightarrow q$	Given
2. $p \rightarrow q \wedge q \rightarrow p$	Equivalence Rule
3. $\neg p \vee q \wedge \neg q \vee p$	Implication $\times 2$
4. $q \vee \neg p \wedge p \vee \neg q$	Commutative Law
5. $\neg\neg q \vee \neg p \wedge \neg\neg p \vee \neg q$	Double Negation
6. $\neg q \rightarrow \neg p \wedge \neg p \rightarrow \neg q$	Implication $\times 2$
7. $\neg p \rightarrow \neg q \wedge \neg q \rightarrow \neg p$	Commutativity
8. $\neg p \leftrightarrow \neg q$	Equivalence Rule

  - Thus we have shown that the two are equivalent.
- (Last 10 minutes) Quiz.