Recitation 13

Taylor Spangler

April 16, 2012

- (3 min max) Questions about last week's quiz?
- Questions about lecture / homework so far?
- Today, Asymptotics and Summations
- The following is problem 3.2:25 – Give a good big-O estimate for the following:
  - $(n^2+8)(n+1)$, well we can see that this becomes n^3+n^2+8n+1...this is clearly $O(n^3)$
  - $(n \log n + n^2)(n^3+1)$, becomes $n^5 + n^4 \log n + n^2 + n \log n$, again easily this is $O(n^5)$
  - $(n!+2^n)(n^3+\log(n^2+1))$, becomes $n! * n^3 + n! * \log(n^2+1) + 2^n * n^3 + 2^n * \log(n^2+1)$. Well, this one is a little bit trickier, any guesses? It is actually $O(n! * n^3)$, can anybody tell me why it's not $O(n!)$ or 2^n
- Now, problem 3.2:31 – Show that $f(x) \in \Theta(g(x))$ iff $f(x) \in O(g(x))$ and $g(x) \in \Theta(f(x))$.
  1. $\Rightarrow$ First we begin with a definition: $f(x) \in \Theta(g(x))$ if $f(x) \in O(g(x))$ and $f(x) \in \Omega(g(x))$
     - So we can see that $\exists c$ such that $f(x) \leq c * g(x)$. We also know that $f(x) \geq c * g(x)$.
     - Reversing the second inequality we get $g(x) \leq c * f(x)$ (really $\frac{1}{c}$, but that is also a constant so we'll just use c here for purposes of simplicity.
     - So then we have $f(x) \in O(g(x))$ and $g(x) \in O(f(x))$
  2. $\Leftarrow$
     - Suppose $f(x) \in O(g(x))$ and $g(x) \in O(f(x))$.
     - Then we know that $f(x) \leq c_1 * g(x)$ and $g(x) \leq c_2 * f(x)$.
     - so $\frac{1}{c_1} g(x) \leq f(x) \leq c_2 * g(x)$.
     - But this is the definition of Θ, therefore $f(x) \in \Theta(g(x))$.

- What is the tightest bound we can form here:
  1. $x^2 + 3x + 5 \in ???(x^3)$: big-O
  2. $2^n \log(6) + n^2 \in ???(2^n)$: Θ
  3. $2^n * n! + 2^n \log(n) \in ???(2^n)$: Ω
- Why don't we prove the first one of the previous problem? Let's use the limit method
  - $\lim_{x\to\infty} \frac{x^2+3x+4}{x^3} = \lim_{x\to\infty} \frac{2x+3}{3x^2} = \lim_{x\to\infty} \frac{2}{6x} = 0$
  - Therefore we can conclude that x^3 grows much faster, and so we get that $x^2 + 3x + 4 \in O(x^3)$
- Next up Sequences: Can we name the first 4 terms of the following sequence?
  1. $a_0 = 2^0 + 1 = 1$
  2. $a_1 = 2^1 + 1 = 3$
  3. $a_2 = 2^2 + 1 = 5$
  4. $a_3 = 2^3 + 1 = 9$
- Now compute the following sum $\displaystyle\sum_{i=1}^{5} 6$
  - Well clearly this is just 6+6+6+6+6 = 6*5 = 30
- How about the following geometric $\displaystyle\sum_{i=1}^{8} 3 * 2^i$
  - Well here we can recognize that this is a geometric sum $\displaystyle\sum_{i=0}^{n} ar^i$ only here we start at 1 instead of zero, so we can simply compute it by subtracting the first term from the sum.
  - The formula for computing this is $\frac{a*r^{n+1}-a}{r-1}$, here $r = 2$ and $a = 3$
  - Using the formula we can get $\frac{3*2^9-3}{1}$ = 3*512 - 3 = 1533
  - But remember we have to subtract the first term, so 1533 - $3 * 2^0$ = 1530.