Recitation 13
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April 16, 2012

• (3 min max) Questions about last week’s quiz?
• Questions about lecture / homework so far?
• Today, Asymptotics and Summations

The following is problem 3.2:25 – Give a good big-\(O\) estimate for the following:

- \((n^2 + 8)(n + 1)\), well we can see that this becomes \(n^3 + n^2 + 8n + 1\)...this is clearly \(O(n^3)\)
- \((n \log n + n^2)(n^3 + 1)\), becomes \(n^5 + n^4 \log n + n^2 + n \log n\), again easily this is \(O(n^5)\)
- \((n! + 2^n)(n^3 + \log(n^2 + 1))\), becomes \(n!n^3 + n! \log(n^2 + 1) + 2^n n^3 + 2^n \log(n^2 + 1)\). Well, this one is a little bit trickier, any guesses? It is actually \(O(n!n^3)\), can anybody tell me why it’s not \(O(n!)\) or \(2^n\)

Now, problem 3.2:31 – Show that \(f(x) \in \Theta(g(x))\) iff \(f(x) \in O(g(x))\) and \(g(x) \in O(f(x))\).

1. \(\Rightarrow\) First we begin with a definition: \(f(x) \in \Theta(g(x))\) if \(f(x) \in O(g(x))\) and \(f(x) \in \Omega(g(x))\)
   - So we can see that \(\exists c\) such that \(f(x) \leq c \cdot g(x)\). We also know that \(f(x) \geq c \cdot g(x)\).
   - Reversing the second inequality we get \(g(x) \leq c \cdot f(x)\) (really \(\frac{1}{c}\), but that is also a constant so we’ll just use \(c\) here for purposes of simplicity.
   - So then we have \(f(x) \in O(g(x))\) and \(g(x) \in O(f(x))\)

2. \(\Leftarrow\)
   - Suppose \(f(x) \in O(g(x))\) and \(g(x) \in O(f(x))\).
   - Then we know that \(f(x) \leq c_1 \cdot g(x)\) and \(g(x) \leq c_2 \cdot f(x)\).
   - So \(\frac{1}{c_1} g(x) \leq f(x) \leq c_2 \cdot g(x)\).
   - But this is the definition of \(\Theta\), therefore \(f(x) \in \Theta(g(x))\).
What is the tightest bound we can form here:

1. \( x^2 + 3x + 5 \in \Omega(x^3) \): big-O
2. \( 2^n \log(6) + n^2 \in \Theta(2^n) \)
3. \( 2^n * n! + 2^n \log(n) \in \Omega(2^n) \)

Why don’t we prove the first one of the previous problem? Let’s use the limit method

\[
\lim_{x \to \infty} \frac{x^2 + 3x + 4}{x^3} = \lim_{x \to \infty} \frac{2x + 3}{3x^2} = \lim_{x \to \infty} \frac{2}{6x} = 0
\]

Therefore we can conclude that \( x^3 \) grows much faster, and so we get that \( x^2 + 3x + 4 \in O(x^3) \)

Next up Sequences: Can we name the first 4 terms of the following sequence?

1. \( a_0 = 2^0 + 1 = 1 \)
2. \( a_1 = 2^1 + 1 = 3 \)
3. \( a_2 = 2^2 + 1 = 5 \)
4. \( a_3 = 2^3 + 1 = 9 \)

Now compute the following sum \( \sum_{i=1}^{6} 6 \)

- Well clearly this is just \( 6+6+6+6+6 = 6*5 = 30 \)

How about the following geometric \( \sum_{i=1}^{8} 3 * 2^i \)

- Well here we can recognize that this is a geometric sum \( \sum_{i=0}^{n} ar^i \) only here we start at 1 instead of zero, so we can simply compute it by subtracting the first term from the sum.
- The formula for computing this is \( \frac{ar^{n+1} - a}{r-1} \), here \( r = 2 \) and \( a = 3 \)
- Using the formula we can get \( \frac{3*2^9 - 3}{1} = 3*512 - 3 = 1533 \)
- But remember we have to subtract the first term, so \( 1533 - 3*2^0 = 1530 \).