Algorithms: An Introduction

‘Algorithm’ is a distortion of Al-Khawarizmi, a Persian mathematician

Section 3.1 of Rosen
Spring 2012
CSCE 235 Introduction to Discrete Structures
Course web-page: cse.unl.edu/~cse235
Questions: Piazza
Outline

• Introduction & definition
• Algorithms categories & types
• Pseudo-code
• Designing an algorithm
  – Example: MAX
• Greedy Algorithms
  – CHANGE
Computer Science is About Problem Solving

• **A Problem** is specified by

  1. **The givens** (a formulation)
     • A set of objects
     • Relations between them

  2. **The query**
     • The information one wants to extract from the formulation, the question to answer

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<tr>
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• **An algorithm** is a method or procedure that solves *instances* of a problem
 Algorithms: Formal Definition

• **Definition**: An algorithm is a sequence of unambiguous instructions for solving a problem.

• Properties of an algorithm
  – **Finite**: the algorithm must eventually terminate
  – **Complete**: Always give a solution when one exists
  – **Correct (sound)**: Always give a correct solution

• For an algorithm to be an acceptable solution to a problem, it must also be **effective**. That is, it must give a solution in a ‘reasonable’ amount of time

• **Efficient**: runs in polynomial time. Thus, **effective ≠ efficient**

• There can be many algorithms to solve the same problem
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Algorithms: General Techniques

• There are many broad categories of algorithms
  – Deterministic versus Randomized (e.g., Monte-Carlo)
  – Exact versus Approximation
  – Sequential/serial versus Parallel, etc.

• Some general styles of algorithms include
  – Brute force (enumerative techniques, exhaustive search)
  – Divide & Conquer
  – Transform & Conquer (reformulation)
  – Greedy Techniques
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Good Pseudo-Code: Example

**INTERSECTION**

*Input:* Two finite sets $A$, $B$

*Output:* A finite set $C$ such that $C = A \cap B$

1. $C \leftarrow \emptyset$
2. If $|A| > |B|$
3. Then $\text{SWAP}(A,B)$
4. End
5. For every $x \in A$ Do
6. If $x \in B$
7. Then $C \leftarrow C \cup \{x\}$

8. End
9. End
10. Return $C$
Algorithms: Pseudo-Code

- Algorithms are usually presented using **pseudo-code**
- **Bad pseudo-code**
  - gives too many details or
  - is too implementation specific (i.e., actual C++ or Java code or giving every step of a sub-process such as set union)
- **Good pseudo-code**
  - Is a balance between clarity and detail
  - Abstracts the algorithm
  - Makes good use of mathematical notation
  - Is easy to read and
  - Facilitates implementation (reducible, does not hide away important information)
Writing Pseudo-Code: Advice

• Input/output must properly defined
• All your variables must be properly initialized, introduced
• Variables are instantiated, assigned using $\leftarrow$
• All `commands' (while, if, repeat, begin, end) bold face \bf

For $i \leftarrow 1$ to $n$ Do

• All functions in small caps UNION($s,t$) \sc
• All constants in courier: $\pi \leftarrow 3.14$ \tt
• All variables in italic: $temperature \leftarrow 78$ (\it, \em)
• LaTeX: Several algorithm formatting packages exist on WWW
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Designing an Algorithm

- A general approach to designing algorithms is as follows
  - Understanding the problem, **assess its difficulty**
  - Choose an approach (e.g., exact/approximate, deterministic/probabilistic)
  - (Choose appropriate data structures)
  - Choose a strategy
  - Prove
    1. Termination
    2. Completeness
    3. Correctness/soundness
  - Evaluate complexity
  - Implement and test it
  - Compare to other known approach and algorithms
Algorithm Example: MAX

• When designing an algorithm, we usually give a formal statement about the problem to solve

• Problem
  – **Given**: a set $A=\{a_1, a_2, \ldots, a_n\}$ of integers
  – **Question**: find the index $i$ of the maximum integer $a_i$

• A straightforward idea is
  – Simply store an initial maximum, say $a_1$
  – Compare the stored maximum to every other integer in $A$
  – Update the stored maximum if a new maximum is ever encountered
Pseudo-code of Max

MAX

Input: A finite set $A=\{a_1, a_2, \ldots, a_n\}$ of integers

Output: The largest element in the set

1. $temp \leftarrow a_1$
2. For $i = 2$ to $n$ Do
3. If $a_i > temp$
4. Then $temp \leftarrow a_i$
5. End
6. End
7. Return $temp$
Algorithms: Other Examples

- Check Bubble Sort and Insertion Sort in your textbooks
- ... which you should have seen ad nauseum in CSE 155 and CSE 156
- And which you will see again in CSE 310
- Let us know if you have any questions
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Greedy Algorithms

• In many problems, we wish to not only find a solution, but to find the best or optimal solution

• A simple technique that works for some optimization problems is called the greedy technique

• As the name suggests, we solve a problem by being greedy:
  – Choose what appears now to be the best choice
  – Choose the most immediate best solution (i.e., think locally)

• Greedy algorithms
  – Work well on some (simple) algorithms
  – Usually they are not guaranteed to produce the best globally optimal solution
Change-Making Problem

• We want to give change to a customer but we want to minimize the number of total coins we give them

• Problem
  – Given: An integer \( n \) an a set of coin denominations \((c_1, c_2, ..., c_r)\) with \( c_1 > c_2 > ... > c_r \)
  – Query: Find a set of coins \( d_1, d_2, ..., d_k \) such that \( \sum_{i=1}^{k} d_i = n \) and \( k \) is minimized
Greedy Algorithm: CHANGE

**CHANGE**

**Input:** An integer $n$ and a set of coin denominations $\{c_1, c_2, \ldots, c_r\}$ with $c_1 > c_2 > \ldots > c_r$

**Output:** A set of coins $d_1, d_2, \ldots, d_k$ such that $\sum_{i=1}^{k} d_i = n$ and $k$ is minimized

1. $C \leftarrow \emptyset$
2. For $i = 1$ to $r$ Do
3.  While $n \geq c_i$ Do
4.   $C \leftarrow C \cup \{c_i\}$
5.  $n \leftarrow n - c_i$
6.  End
7. Return $C$
CHANGE: Analysis (1)

• Will the algorithm always produce an optimal answer?
• Example
  – Consider a coinage system where \( c_1=20, c_2=15, c_3=7, c_4=1 \)
  – We want to give 22 ‘cents’ in change
• What is the output of the algorithm?
• Is it optimal?
  • It is not optimal because it would give us two \( c_4 \) and one \( c_1 \) (3 coins). The optimal change is one \( c_2 \) and one \( c_3 \) (2 coins)
CHANGE: Analysis (2)

• What about the US currency system: is the algorithm correct in this case?
• Yes, in fact it is. We can prove it by contradiction.
• For simplicity, let us consider
  \[ c_1=25, \ c_2=10, \ c_3=5, \ c_4=1 \]
Optimality of **CHANGE** (1)

- Let $C = \{d_1, d_2, \ldots, d_k\}$ be the solution given by the greedy algorithm for some integer $n$.
- By way of contradiction, assume there is a better solution $C' = \{d'_1, d'_2, \ldots, d'_l\}$ with $l < k$.
- Consider the case of quarters. Say there are $q$ quarters in $C$ and $q'$ in $C'$.
  1. If $q' > q$, the greedy algorithm would have used $q'$ by construction. Thus, it is impossible that the greedy uses $q < q'$.
  2. Since the greedy algorithms uses as many quarters as possible, $n = q(25) + r$, where $r < 25$. If $q' < q$, then, $n = q'(25) + r'$ where $r' \geq 25$. $C'$ will have to use more smaller coins to make up for the large $r'$. Thus $C'$ is not the optimal solution.
  3. If $q = q'$, then we continue the argument on the smaller denomination (e.g., dimes). Eventually, we reach a contradiction.
- Thus, $C = C'$ is our optimal solution
Optimality of CHANGE (2)

- But, how about the previous counterexample? Why (and where) does this proof?

- We need the following lemma:
  If $n$ is a positive integer, then $n$ cents in change using quarters, dimes, nickels, and pennies using the fewest coins possible
  - Has at most two dimes,
  - Has at most one nickel
  - Has at most four pennies, and
  - Cannot have two dimes and a nickel
  
  The amount of change in dimes, nickels, and pennies cannot exceed 24 cents
Greedy Algorithm: Another Example

• Check the problem of Scenario I, page 25 in the slides IntroductiontoCSE235.ppt
• We discussed then (remember?) a greedy algorithm for accommodating the maximum number of customers. The algorithm
  – terminates, is complete, sound, and satisfies the maximum number of customers (finds an optimal solution)
  – runs in time linear in the number of customers
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