Week 5 Recitation

Robert Woodward

February 6, 2011

- (3 min max) Go over quiz from last week
- (3 min max) Go over homework from last week.
- Questions about lecture / homework so far?
- 1.6:23) Show that at least 10 of any 64 days chosen must fall on the same day of the week.
  - We proceed with a proof by contradiction.
  - We assume that it is not the case that "at least 10 days of any 64 days chosen fall on the same day of the week."
  - This means that "less than 10 days of any 64 days chosen fall on the same day of the week." Assume, it is 9 days. Then we have "9 days of any 64 days chosen fall on the same day of the week."
  - There are 7 days in a week. If we choose 9 days out of every day of the week, then we have chosen $9\times$ 7days = 63 days, exactly.
  - However, we need to choose 64 days. Therefore, there is one extra day that needs to be chosen. Whichever day we choose, this choice will bump up the 'count' of chosen days to 10, which contradicts the statement "9 days of any 64 days chosen fall on the same day of the week."
  - Therefore, at least 10 of any 64 days chosen must fall on the same day of the week.
- Example from: http://marauder.millersville.edu/ bikenaga/mathproof/rules-of-inference/rules-of-inference.html

  Given:

  1. $p \wedge q$
  2. $p \rightarrow \neg(q \wedge r)$
  3. $s \rightarrow r$

Prove $\neg s$ using the rules of inference.

	Step	**Reason**
1.	$p \wedge q$	Premise
2.	$p \to \neg(q \wedge r)$	Premise
3.	$s \to r$	Premise
4.	p	Simplification of (1)
5.	q	Simplification of (1)
6.	$\neg(q \wedge r)$	Modus Ponens (2) and (4)
7.	$\neg q \vee \neg r$	DeMorgan's Law (6)
8.	$\neg r$	Disjunctive syllogism (5) and (7)
9.	$\neg s$	Modus Tollens (3) and (8)

- 1.5:15a) Given

  1. All students in this class understand logic.
  2. Xavier is a student in this class.

  Prove that, Xavier understands logic.

  - Define your predicates:
    - $Q(x)$: x is in the class
    - $P(x)$: x understands logic
  - Universe of Discourse: All students
  - Theory:
    1. $\forall x P(x) \to Q(x)$
    2. $P(Xavier)$
  - We want to prove : $Q(Xavier)$
  - Proof:

	Step	**Reason**
1.	$\forall x P(x) \to Q(x)$	Premise
2.	$P(Xavier)$	Premise
3.	$P(Xavier) \to Q(Xavier)$	Universal instantiation from (1)
4.	$Q(Xavier)$	Modus ponens from (2) and (3)

- 1.6:27) Prove that for every n a positive integer, n is odd *if and only if* $5n+6$ is odd.

  $\Rightarrow$ First, we prove: n is odd $\Rightarrow 5n+6$ is odd

  1. n is odd $\Rightarrow n = 2k+1$ *by definition*
  2. Plugging in the value of n into $5n+6$, $5n+6 = 5(2k+1)+6 = 10k+5+6 = 10k+11 = 2(5k+2)+1 = 2k'+1$
  3. Therefore, $5n+6$ is odd. $\square$

$\Leftarrow$ Next we prove: $5n + 6$ odd $\Rightarrow n$ is odd. We use a proof by contrapositive and establish that "n is even $\Rightarrow 5n + 6$ even."

1. n is even $\Rightarrow n = 2k$ *by definition*
2. Plugging in the value of n into $5n + 6 = 5(2k) + 6 = 10k + 6 = 2(5k + 3) = 2k'$
3. Therefore, $5n + 6$ is even. $\square$

Therefore, n is odd iff $5n + 6$ is odd. $\square$

- (Last 12 minutes) Give quiz