Week 4 Recitation

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- (3 min max) Go over quiz from last week
- (3 min max) Go over homework from last week.
- Questions about lecture / homework so far?
- Guidance for making proofs using equivalence rules:
  1. Write the equivalence to prove.
  2. Start from either the LHS (left hand-side) or the RHS, whichever you are more comfortable with or whichever seems more complex (so you can simplify it).
  3. Put a number on each step starting from zero.
  4. Make sure to put the equivalence sign between a step and the next to clarify the meaning of the transition.
  5. Justify each transition with the name of the equivalence law that you have applied.
  6. Further, most of the time, you want to proceed through the following sequence of operations, some of them may be ommitted: Remove biconditionals, remove implications, push negation inwards (applying DeMorgan laws or double negation), apply distribution laws. Then, you may have to use commutativity or associativity before reconfiguring what you have so far in the format of the 'goal' expression. The best method is *practice*.
- Rosen 1.2:19
  Show that $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

Step	**Sentence**	**Equivalence law**
0	$\neg p \leftrightarrow q$	
1	$\equiv (\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$	Biconditional law
2	$\equiv (p \vee q) \wedge (\neg q \vee \neg p)$	Implication law
3	$\equiv (q \vee p) \wedge (\neg p \vee \neg q)$	Commutative law
4	$\equiv (\neg q \rightarrow p) \wedge (p \rightarrow \neg q)$	Implication
5	$\equiv p \leftrightarrow \neg q$	Biconditional law.

Notice that, above, we used the implication rule. Using the contrapositive is also an option:

Step	**Sentence**	**Equivalence law**
0	$\neg p \leftrightarrow q$	
1	$\equiv (\neg p \to q) \wedge (q \to \neg p)$	Biconditional law
2	$\equiv (\neg q \to \neg\neg p) \wedge (\neg\neg p \to \neg q)$	Contrapositive on both conjuncts.
3	$\equiv (\neg q \to p) \wedge (p \to \neg q)$	Double negation, applied twice.
4	$\equiv p \leftrightarrow \neg q$	Biconditional law.

The proofs are quite similar because $(p \to q) \equiv (\neg q \to \neg p) \equiv (\neg p \vee q)$.

- Politicians can fool some of the people all of the time, all of the people some of the time, but they cannot fool all of the people all of the time

  1. Predicates and their meaning:
     - $Fools(x, y, t)$: x fools y at time t
     - $P(x)$: x is a politician
  2. Universe of discourse: x, y all human beings, t all time instants
  3. $\forall x[P(x) \to [(\exists y \forall t Fools(x, y, t)) \wedge (\exists t \forall y Fools(x, y, t)) \wedge \neg(\forall y \forall t Fools(x, y, t))]]$

  The expression $\neg(\forall y \forall t Fools(x, y, t))$ can be difficult to spell out in English. We recommand that you use the expression "*it is not the case that*" whenever an expression starts with a negation. Thus, the above reads: "It is not the case that every x fools every y at every time t."

- Beware of errors:
  - Let's compare the meaning of the two expressions:
    * $\exists t \forall y Fools(x, y, t)$: There is a least one time (or one incident) where everyone was fooled by x.
    * $\forall y \exists t Fools(x, y, t)$: Everyone is fooled at some point in time by x, but the time is not necessarily the same for all human beings.

    Naturally, we intend the first meaning.
  - Also, compare the meaning of the following two expressions:
    * $\forall x[P(x) \to \neg(\forall y \forall t Fools(x, y, t))]$
      $\equiv \forall x[P(x) \to (\exists y \exists t \neg Fools(x, y, t))]$: It is not the case that every x fools every y at every time t.
    * $\forall x[P(x) \to (\forall y \forall t \neg Fools(x, y, t))]$: Politicians do not ever fool anyone.

- Every student enrolled in CSE235 knows LaTeX but only some students use LaTeX in their homework.

1. Predicates and their meaning:
    - $Enrolled(x, y)$: x is enrolled in class y.
    - $WritesHomework(x, y, z)$: x writes homework for y in z.
    - $Knows(x, y)$: x knows language y,
2. Universe of discourse: all students, classes, languages
3. $(\forall x Enrolled(x, CSE235) \rightarrow (Knows(x, \text{\LaTeX})))\wedge$
   $(\exists x Enrolled(x, CSE235) \wedge WritesHomework(x, CSE235, \text{\LaTeX}))$

- Every student enrolled in CSE235 knows LaTeX but only some students use LaTeX in their homework. However, every student in CSE310 uses LaTeX in his/her homework.

1. Predicates and their meaning: Use the same as above
2. Universe of discourse: Same as above
3. $(\forall x Enrolled(x, CSE235) \rightarrow (Knows(x, \text{\LaTeX})))\wedge$
   $(\exists x Enrolled(x, CSE235) \wedge WritesHomework(x, CSE235, \text{\LaTeX}))\wedge$
   $(\forall x Enrolled(x, CSE310) \rightarrow (WritesHomework(x, CSE310, \text{\LaTeX}))$

- Every student enrolled in CSE235 knows some language because they took a course about it

1. Predicates and their meaning:
    - $Enrolled(x, y)$: x is enrolled in class y
    - $Knows(x, y)$: x knows language y,
    - $TC(x, y)$: x took course about language y
2. Universe of discourse: x all students, y all languages
3. $\forall x \exists y Enrolled(x, CSE235) \rightarrow (Knows(x, y) \wedge TC(x, y))$

- Rosen 1.2:21
  Show that $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$

Step	**Sentence**	**Equivalence law**
0.	$\neg(p \leftrightarrow q)$	
1.	$\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$	Biconditional law
2.	$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p))$	Implication law
3.	$\equiv \neg((\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p))$	Distributive law
4.	$\equiv \neg((\neg p \wedge \neg q) \vee (q \wedge p))$	Identity law
5.	$\equiv (p \vee q) \wedge (\neg q \vee \neg p)$	DeMorgan's law
6.	$\equiv (\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$	Implication law
7.	$\equiv \neg p \leftrightarrow q$	Biconditonal law

- Rosen 1.2:23
  Show that $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$

Step	**Sentence**	**Equivalence law**
0.	$(p \to r) \land (q \to r)$	
1.	$\equiv (\neg p \lor r) \land (\neg q \lor r)$	Implication law
2.	$\equiv (\neg p \land \neg q) \lor r$	Distributive law
3.	$\equiv \neg(\neg p \land \neg q) \to r$	Implication law
4.	$\equiv (p \lor q) \to r$	DeMorgan's law and double negation law

- Rosen 1.2:15
  Show that $\neg q \land (p \to q)) \to \neg p$ is a tautology

Step	**Sentence**	**Equivalence law**
0.	$\neg(\neg q \land (p \to q)) \lor \neg p$	
1.	$\equiv \neg(\neg q \land (\neg p \lor q)) \lor \neg p$	Implication law
2.	$\equiv (q \lor (p \land \neg q)) \lor \neg p$	DeMorgan's law
3.	$\equiv ((q \lor p) \land (q \lor \neg q)) \lor \neg p$	Distributive law
4.	$\equiv ((q \lor p)) \lor \neg p$	Identity law
5.	$\equiv (q \lor (p \lor \neg p))$	Associative law
6.	$\equiv q \lor T$	Identity law
7.	$\equiv T$	Identity law

- (Last 10 minutes) Give quiz