Week 3 Recitation

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- Questions about lecture / homework so far?
- A challenging question: Rosen 1.1, number 63 (Page 21).

  1. First, we will set up our *terms*:
     - Let b be the butler
     - Let c be the cook
     - Let g be the gardener
     - Let h be the handyman

     We will have a 0 represent they are lying, and a 1 represent they are telling the truth.
  2. Next we will parse the question to get our *clauses*:
     - "If the butler is telling the truth then so is the cook:" $b \to c$
     - "The cook and the gardener cannot both be telling the truth:" $\neg(c \wedge g) \equiv \neg c \vee \neg g$
     - "The gardener and the handyman are not both lying:" $\neg(\neg g \wedge \neg h) \equiv g \vee h$
     - "If the handyman is telling the truth then the cook is lying:" $h \to \neg c$
  3. Now, we want to find a *model* for our *sentence*.

b	c	g	h	$b \to c$	$\neg c \lor \neg g$	$g \lor h$	$h \to \neg c$
0	0	0	0	1	1	0	-
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	-
0	1	0	1	1	1	1	0
0	1	1	0	1	0	-	-
0	1	1	1	1	0	-	-
1	0	0	0	0	-	-	-
1	0	0	1	0	-	-	-
1	0	1	0	0	-	-	-
1	0	1	1	0	-	-	-
1	1	0	0	1	1	0	-
1	1	0	1	1	1	1	0
1	1	1	0	1	0	-	-
1	1	1	1	1	0	-	-

Since we are looking for a *model*, an assignment of values to our terms such that all of clauses are true, we can stop looking at an assignment once one of the clauses evaluates to true. Because of the stops, '-' is inserted into the truth table when no evaluation is needed.

4. Based on our three models, we can conclude that the butler and the cook must be lying and either the gardener, the handyman, or both of them are telling the truth.

- Similar to Problem A of Homework 1: "Suppose that $a \land b$ is known to be true".

  What is $a \lor b$ and what is $a \to b$?

  First, draw the truth table for $a \land b$:

a	b	$a \land b$
0	0	0
0	1	0
1	0	0
1	1	1

  Notice, there is only one *model* where $a \land b$ is true, when $a \leftarrow 1$ and $b \leftarrow 1$.

  Now knowing a, b, find $a \lor b$:

a	b	$a \lor b$
0	0	0
0	1	1
1	0	1
1	**1**	**1**

Therefore, $a \vee b$ is 1 because $a \leftarrow 1$ and $b \leftarrow 1$.

Now find $a \rightarrow b$:

a	b	$a \vee b$
0	0	1
0	1	1
1	0	0
1	**1**	**1**

- Review of SAT: $(a \vee b \vee \neg c \vee \neg d) \wedge (\neg b \vee c) \wedge (\neg a \vee c \vee d)$
  - What is a *term*?
  - What is a *literal*?
  - What is a *clause*?
  - What is a *model*?

a	b	c	d	$A : (a \vee b \vee \neg c \vee \neg d)$	$B : (\neg b \vee c)$	$C : (\neg a \vee c \vee d)$	$(A) \wedge (B) \wedge (C)$
0	0	0	0	1	1	1	1
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	1	0	1	0
0	1	0	1	1	0	1	0
0	1	1	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	0	0
1	0	0	1	1	1	1	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	1	0
1	1	1	0	1	1	1	1
1	1	1	1	1	1	1	1

- Go over 1.1:23 (a) in text.
  - Let p be "It snows today" and let q be "I will ski tomorrow".
  - Converse is $q \rightarrow p$
    "I will ski tomorrow only if it snows today"

- Contrapositive is $\neg q \to \neg p$
  "If I do not ski tomorrow then it will not have snowed today"
- Inverse is $\neg p \to \neg q$
  "If it does not snow today I will not ski tomorrow"

- Draw truth tables for the converse, contrapositive, and inverse. What can we conclude about contrapositive?

p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg q \to \neg p$	$\neg p \to \neg q$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	1	0
1	0	0	1	0	1	0	1
1	1	1	1	0	0	1	1

- 1.1:7 f in text. $(p \vee q) \wedge (p \to \neg q)$
- 1.1:33 (f) in text.

p	q	r	$A : \neg p \leftrightarrow \neg q$	$B : q \leftrightarrow r$	$A \leftrightarrow B$
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	0	1
0	1	1	0	1	0
1	0	0	0	1	0
1	0	1	0	0	1
1	1	0	1	0	0
1	1	1	1	1	1

- 1.1:13 (a) in text.
  - Let p stand for $1 + 1 = 2$
  - Let q stand for $2 + 2 = 5$
  - What is $p \to q$? $1 \to 0$ which is false.
- (Last 10 minutes) Give quiz