Week 15 Recitation

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- Questions about lecture / homework so far?
- Show how to do summation in Maple
  1. SSH into cse.unl.edu
  2. Type in "maple" to launch Maple
  3. Type in "sum(*What to Sum*, *Over what range*);". For example, to sum $\sum_{k=1}^{10} k^2$, you would type "sum(k^2, k = 1..10);". Another example, to sum $\sum_{k=1}^{n} k^2$, you would type "sum(k^2,k = 1..n);".
- (Similar to Rosen 2.4, problem 5a) Write the general summation expression of:

  > the sum of the first n numbers that begins with 2 and in which each successive term is 3 more than the preceding term

  First, recall that: $\sum_{i=m}^{n} i = \frac{(n-m+1)(n+m)}{2}$, and $\sum_{i=m}^{n} 1 = n - m + 1$.

  First we can write out the sequence to get an idea of what it will look like:

$$\{2, 5, 8, 11, 14, 17, 20, \ldots\}.$$

  This sequence is an *arithmetic progression* and is of the form $a+nd$, where a is the initial term and d is the common difference. Therefore, the *closed form* of the summation, or series, is:

$$\begin{aligned}
\sum_{i=0}^{n-1}(2 + i \times 3) &= \sum_{i=0}^{n-1}(2 + 3i) = 2\sum_{i=0}^{n-1} 1 + 3\sum_{i=0}^{n-1} i \\
&= 2((n-1) - 0 + 1) + 3\frac{(n-1)((n-1)+1)}{2} \\
&= 2n + \frac{3}{2}n(n-1) \\
&= \frac{n(3n+1)}{2}
\end{aligned}$$

- (Rosen 2.4.19) Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers. This type of sum is called telescoping.

  We can write out this summation as:

  $$(a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + \ldots + (a_{n-1} - a_{n-2}) + (a_n - a_{n-1})$$

  Notice what we can cancel out a_1 and $-a_1$, a_2 and $-a_2$ all the way up to a_{n-1} and $-a_{n-1}$. This leaves us with $-a_0$ and a_n, which is $a_n - a_0$.

- (Similar to Rosen 2.4, Problem 17(a), Page 162): Compute $\sum_{i=1}^{n}\sum_{j=i}^{m}(i+j)$

  Recall that $\sum_{k=m}^{n} k^2 = -\frac{1}{6}(m-n-1)(2m^2+2mn-m+2n^2+n)$, alternatively, $\sum_{k=1}^{m} k^2 = \frac{n(n+1)(2n+1)}{6}$.

  $$\begin{aligned}
  \sum_{i=1}^{n}\sum_{j=i}^{m}(i+j) &= \sum_{i=1}^{n}(\sum_{j=i}^{m} i + \sum_{j=i}^{m} j) && (1)\\
  &= \sum_{i=1}^{n}(i\sum_{j=i}^{m} 1 + \sum_{j=i}^{m} j) && (2)\\
  &= \sum_{i=1}^{n}(i \times (m-i+1) + \frac{(m-i+1)(m+i)}{2}) && (3)\\
  &= \sum_{i=1}^{n}(m \times i - i^2 + i + \frac{m^2 + m \times i - m \times i - i^2 + m + i}{2}) && (4)\\
  &= \sum_{i=1}^{n}(m \times i - i^2 + i + \frac{m^2}{2} - \frac{i^2}{2} + \frac{m}{2} + \frac{i}{2}) && (5)\\
  &= \sum_{i=1}^{n}(-\frac{3}{2}i^2 + (m+\frac{3}{2})i + (\frac{m^2}{2} + \frac{m}{2})) && (6)\\
  &= -\frac{3}{2}\sum_{i=1}^{n} i^2 + (m+\frac{3}{2})\sum_{i=1}^{n} i + (\frac{m^2}{2}+\frac{m}{2})\sum_{i=1}^{n} 1 && (7)\\
  &= -\frac{3}{2}\frac{n(n+1)(2n+1)}{6} + (m+\frac{3}{2})\frac{(n-1+1)(n+1)}{2}\\
  &\quad +(\frac{m^2}{2}+\frac{m}{2})(n-1+1) && (8)\\
  &= -\frac{3}{2}\frac{n(n+1)(2n+1)}{6} + (m+\frac{3}{2})\frac{n(n+1)}{2} + (\frac{m^2}{2}+\frac{m}{2})n && (9)
  \end{aligned}$$

  - In (1), we split the terms in the inner summation.
  - In (2), we move i outside the second summation because the index of the summation does not depend on i. It is as if i was a constant.

- In (3), we compute the two inner summations.
- In (6), we split the terms in the unique summation left.
- The rest should be straightforward.

- (Rosen 2.4, Problem 17(a), Page 162): Compute $\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j)$

$$\begin{aligned}
\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j) &= \sum_{i=1}^{2}(\sum_{j=1}^{3} i + \sum_{j=1}^{3} j) && (10)\\
&= \sum_{i=1}^{2}(i\sum_{j=1}^{3} 1 + \sum_{j=1}^{3} j) && (11)\\
&= \sum_{i=1}^{2}(i \times (3-1+1) + \frac{3(3+1)}{2}) && (12)\\
&= \sum_{i=1}^{2}(3i+6) && (13)\\
&= \sum_{i=1}^{2} i + 6\sum_{i=1}^{2} 1 && (14)\\
&= 3 \times \frac{2(2+1)}{2} + 6(2+1-1) && (15)\\
&= 3 \times 3 + 6 \times 2 && (16)\\
&= 21 && (17)
\end{aligned}$$

- In (10), we split the terms in the inner summation.
- In (11), we move i outside the second summation because the index of the summation does not depend on i. It is as if i was a constant.
- In (12), we compute the two inner summations.
- In (14), we split the terms in the unique summation left.
- The rest should be straightforward.

- (Last 10 minutes) Quiz