Week 15 Recitation

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• Questions about lecture / homework so far?

• Show how to do summation in Maple

  1. SSH into cse.unl.edu
  2. Type in “maple” to launch Maple
  3. Type in “sum(What to Sum, Over what range);”. For example, to sum \( \sum_{k=1}^{10} k^2 \), you would type “sum(k^2, k = 1..10);”. Another example, to sum \( \sum_{k=1}^{n} k^2 \), you would type “sum(k^2,k = 1..n);”.

• (Similar to Rosen 2.4, problem 5a) Write the general summation expression of:

the sum of the first \( n \) numbers that begins with 2 and in which each successive term is 3 more than the preceding term

First, recall that: \( \sum_{i=m}^{n} i = \frac{(n-m+1)(n+m)}{2} \), and \( \sum_{i=m}^{n} 1 = n - m + 1 \).

First we can write out the sequence to get an idea of what it will look like:

\{2, 5, 8, 11, 14, 17, 20, \ldots\}.

This sequence is an arithmetic progression and is of the form \( a + nd \), where \( a \) is the initial term and \( d \) is the common difference. Therefore, the closed form of the summation, or series, is:

\[
\sum_{i=0}^{n-1} (2 + i \times 3) = \sum_{i=0}^{n-1} (2 + 3i) = 2 \sum_{i=0}^{n-1} 1 + 3 \sum_{i=0}^{n-1} i \\
= 2((n - 1) - 0 + 1) + 3\frac{(n-1)((n-1)+1)}{2} \\
= 2n + \frac{3}{2}n(n-1) \\
= \frac{n(3n+1)}{2}
\]
• (Rosen 2.4.19) Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers. This type of sum is called telescoping.

We can write out this summation as:

$$(a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + \ldots + (a_{n-1} - a_{n-2}) + (a_n - a_{n-1})$$

Notice what we can cancel out $a_1$ and $-a_1$, $a_2$ and $-a_2$ all the way up to $a_{n-1}$ and $-a_{n-1}$. This leaves us with $-a_0$ and $a_n$, which is $a_n - a_0$.

• (Similar to Rosen 2.4, Problem 17(a), Page 162): Compute $\sum_{i=1}^{n} \sum_{j=i}^{m} (i + j)$

Recall that $\sum_{k=m}^{n} k^2 = -\frac{1}{6}(m - n - 1)(2m^2 + 2mn - m + 2n^2 + n)$, alternatively, $\sum_{k=1}^{m} k^2 = \frac{n(n+1)(2n+1)}{6}$.

$$\sum_{i=1}^{n} \sum_{j=i}^{m} (i + j) = \sum_{i=1}^{n} \left( \sum_{j=i}^{m} i + \sum_{j=i}^{m} j \right)$$

$$= \sum_{i=1}^{n} (i \sum_{j=i}^{m} 1 + \sum_{j=i}^{m} j)$$

$$= \sum_{i=1}^{n} (i \times (m - i + 1) + \frac{(m - i + 1)(m + i)}{2})$$

$$= \sum_{i=1}^{n} \left( m \times i - i^2 + i + \frac{m^2 + m \times i - m \times i - i^2 + m + i}{2} \right)$$

$$= \sum_{i=1}^{n} \left( m \times i - i^2 + i + \frac{m^2}{2} - \frac{i^2}{2} + \frac{m}{2} + \frac{i}{2} \right)$$

$$= \sum_{i=1}^{n} \left( -\frac{3}{2} \times i^2 + (m + \frac{3}{2})i + \left( \frac{m^2}{2} + \frac{m}{2} \right) \right)$$

$$= -\frac{3}{2} \sum_{i=1}^{n} i^2 + (m + \frac{3}{2}) \sum_{i=1}^{n} i + \left( \frac{m^2}{2} + \frac{m}{2} \right) \sum_{i=1}^{n} 1$$

$$= -\frac{3}{2} \frac{n(n+1)(2n+1)}{6} + (m + \frac{3}{2}) \frac{n(n-1+1)(n+1)}{2}$$

$$+ \left( \frac{m^2}{2} + \frac{m}{2} \right)(n - 1 + 1)$$

$$= -\frac{3}{2} \frac{n(n+1)(2n+1)}{6} + (m + \frac{3}{2}) \frac{n(n+1)}{2} + \left( \frac{m^2}{2} + \frac{m}{2} \right)n$$

- In (1), we split the terms in the inner summation.
- In (2), we move $i$ outside the second summation because the index of the summation does not depend on $i$. It is as if $i$ was a constant.
– In (3), we compute the two inner summations.
– In (6), we split the terms in the unique summation left.
– The rest should be straightforward.

• (Rosen 2.4, Problem 17(a), Page 162): Compute $\sum_{i=1}^{2} \sum_{j=1}^{3} (i + j)$

\[
\begin{align*}
\sum_{i=1}^{2} \sum_{j=1}^{3} (i + j) &= \sum_{i=1}^{2} (\sum_{j=1}^{3} i + \sum_{j=1}^{3} j) \\
&= \sum_{i=1}^{2} (i \sum_{j=1}^{3} 1 + \sum_{j=1}^{3} j) \\
&= \sum_{i=1}^{2} (i \times (3 - 1 + 1) + \frac{3(3 + 1)}{2}) \\
&= \sum_{i=1}^{2} (3i + 6) \\
&= \sum_{i=1}^{2} i + 6 \sum_{i=1}^{2} 1 \\
&= 3 \times \frac{2(2 + 1)}{2} + 6(2 + 1 - 1) \\
&= 3 \times 3 + 6 \times 2 \\
&= 21
\end{align*}
\]

– In (10), we split the terms in the inner summation.
– In (11), we move $i$ outside the second summation because the index of the summation does not depend on $i$. It is as if $i$ was a constant.
– In (12), we compute the two inner summations.
– In (14), we split the terms in the unique summation left.
– The rest should be straightforward.

• (Last 10 minutes) Quiz