Week 10 Recitation

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- Questions about lecture / homework so far?
- Rosen 8.6.1(a)

  A relation R on a set S is a partial order if it is:

  - Reflexive
  - Antisymmetric
  - Transitive

  Note: That if R satisfies these conditions, the set S with the partial order R is called a *partially ordered set (poset)* and is denoted (S, R).

  Our relation is: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

  Our relation is reflexive, antisymmetric, and transitive. Therefore, our relation is a partial order.

- Rosen 8.6.1 (b)

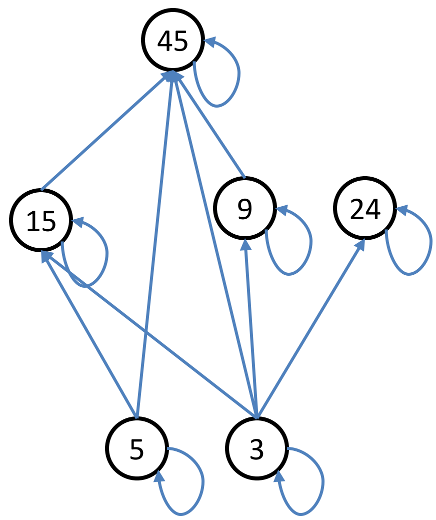
  Our relation is: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & \mathbf{1} \\ 0 & 0 & \mathbf{1} & 1 \end{pmatrix}$

  Our relation is reflexive, and transitive, but is *not* antisymmetric because $(3, 4) \in R \wedge (4, 3) \in R$, but $3 \neq 4$.

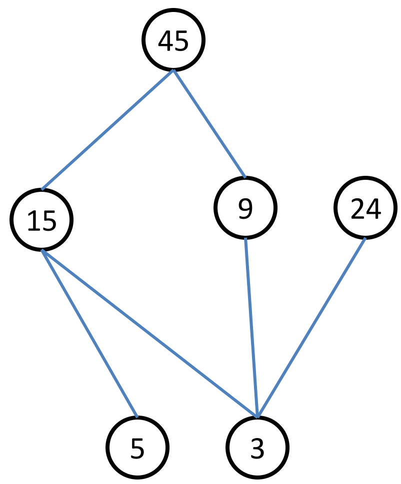
  Therefore, our relation is *not* a partial order.

- Rosen 8.6:33

  First draw the relation as a directed graph, then convert it to a hasse diagram:



Directed Graph



Hasse Diagram

Maximal elements: $\{45, 24\}$

Minimal elements: $\{5, 3\}$

No greatest element

No least element

Upper bounds of $\{3, 5\}$: $\{15, 45\}$

Least upper bounds of $\{3, 5\}$: 15

Lower bounds of $\{15, 45\}$: $\{15, 5, 3\}$

Greatest lower bound of $\{15, 45\}$: 15

Is this poset a latice: No. Consider the lower bounds of $\{5, 3\} = \emptyset$. Therefore, there is no greatest lower bound of $\{5, 3\}$.

> Recall that a poset in which every pair of elements has both a least upper bound and a greatest lower bound is called a latice.

- Rosen 8.1.25(b)

  Complementary relation $\bar{R}$ is the set of ordered pairs $\{(a, b) | (a, b) \notin R\}$.

  $\bar{R} = \{(a, b) | a \text{ does not divide } b\}$

- Consider:

  $$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\bar{R} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

- Rosen 8.3.11 How can the matrix for $\bar{R}$, the complement of the relation R, be found from the matrix representing R, when R is a relation on a finite set A?

  The matrix for $\bar{R}$ can be found from the matrix of R by flipping all the 1's to 0's, and all the 0's to 1's. This result is because any element that was in R is not in $\bar{R}$ ($1 \to 0$), and any element that was not in R is in $\bar{R}$ ($0 \to 1$)

- Quiz (Last 15 minutes)