Title: Solving Problems by Searching
AIMA: Chapter 3 (Sections 3.4)

Introduction to Artificial Intelligence
CSCE 476-876, Spring 2010
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```
function GENERAL-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
     if there are no candidates for expansion then return failure
     choose a leaf node for expansion according to strategy
     if the node contains a goal state then return the corresponding solution
     else expand the node and add the resulting nodes to the search tree
  end
```

Essence of search: which node to expand first?
$\longrightarrow$ search strategy

A strategy is defined by picking the *order of node expansion*

Types of Search

Uninformed: use only information available in problem definition

Heuristic: exploits some knowledge of the domain

Uninformed search strategies

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening depth-first search
6. Bidirectional search

Search strategies

Criteria for evaluating search:

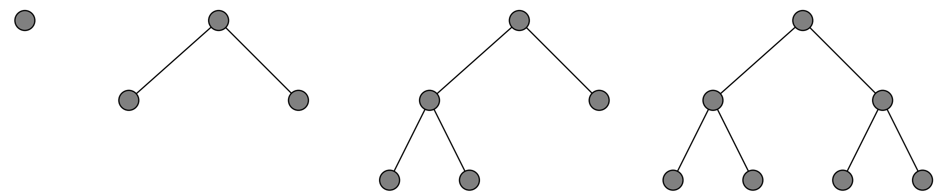
1. Completeness: does it always find a solution if one exists?
2. Time complexity: number of nodes generated/expanded
3. Space complexity: maximum number of nodes in memory
4. Optimality: does it always find a least-cost solution?

Time/space complexity measured in terms of:

- b: maximum branching factor of the search tree
- d: depth of the least-cost solution
- m: maximum depth of the search space (may be ∞)

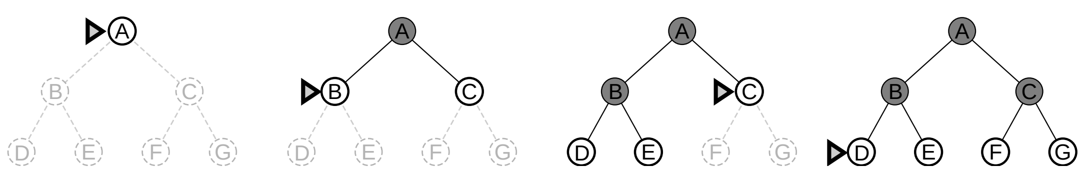
Breadth-first search (I)

→ Expand root node
→ Expand *all* children of root
→ Expand *each* child of root
→ Expand successors of each child of root, etc.



⟶ Expands nodes at depth d before nodes at depth $d + 1$
⟶ Systematically considers all paths length 1, then length 2, etc.
⟶ Implement: put successors at end of queue.. FIFO

Breadth-first search (2)



Breadth-first search (3)

$\longrightarrow$ One solution?

$\longrightarrow$ Many solutions? Finds shallowest goal first

1. Complete? Yes, if b is finite
2. Optimal? provided cost increases monotonically with depth, not in general (e.g., actions have same cost)
3. Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$

   $$O(b^{d+1}) \begin{cases} \text{branching factor } b \\ \text{depth } d \end{cases}$$

4. Space? same, $O(b^{d+1})$, keeps every node in memory, big problem
   can easily generate nodes at 10MB/sec so 24hrs = 860GB

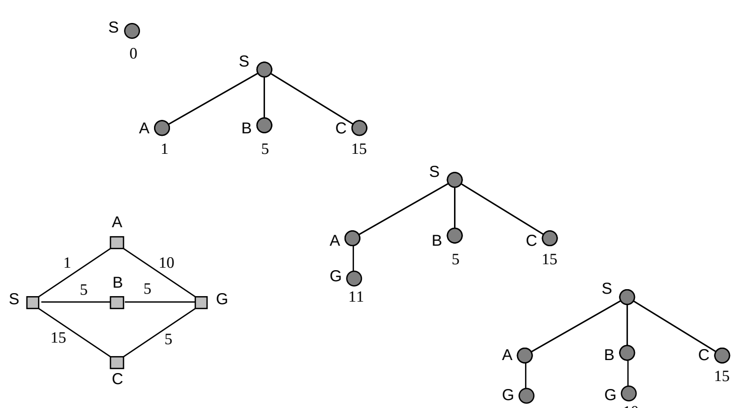
Uniform-cost search (I)

$\longrightarrow$ Breadth-first does not consider <u>path cost</u> $g(x)$

$\longrightarrow$ Uniform-cost expands first lowest-cost node on the fringe

$\longrightarrow$ Implement: sort queue in decreasing cost order

When $g(x) = \text{Depth}(x) \longrightarrow$ Breadth-first $\equiv$ Uniform-cost



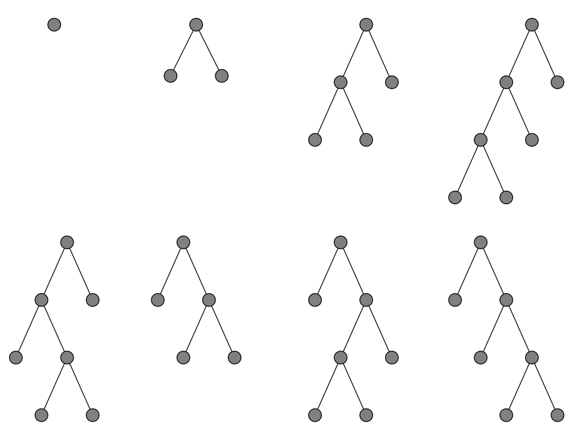
(a) (b)

Uniform-cost search (2)

1. Complete?
   Yes, if cost $\geq \epsilon$
2. Optimal?
   If the cost is a monotonically increasing function
   When cost is added up along path, an operator's cost ....?
3. Time?
   \# of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
   where C^* is the cost of the optimal solution
4. Space?
   \# of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

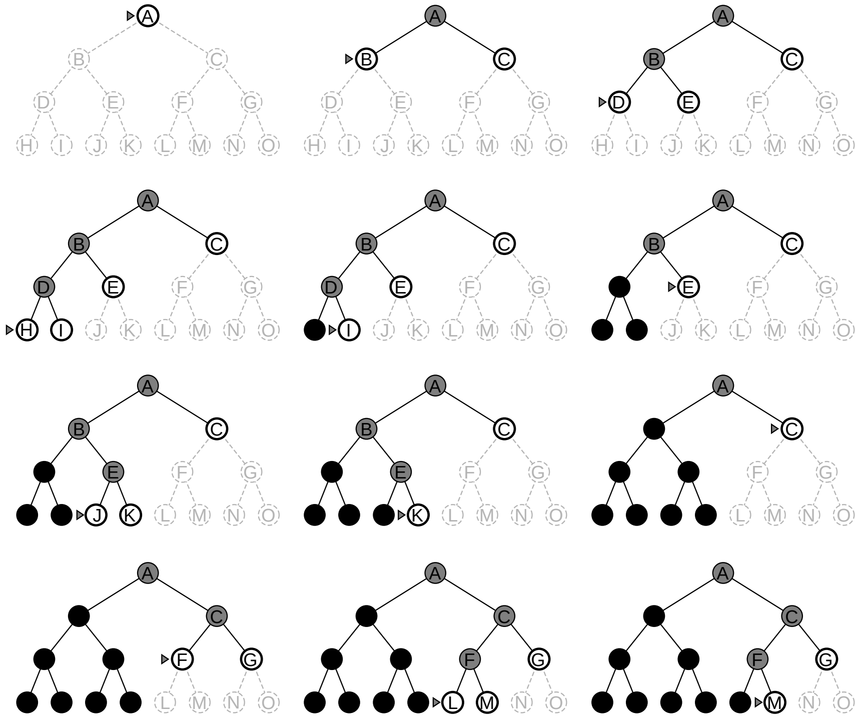
Depth-first search (I)

$\longrightarrow$ Expands nodes at deepest level in tree
$\longrightarrow$ When dead-end, goes back to shallower levels
$\longrightarrow$ Implement: put successors at front of queue.. LIFO



$\longrightarrow$ Little memory: path and unexpanded nodes
For b: branching factor, m: maximum depth, space ....?

Depth-first search (2)



Depth-first search (3)

Time complexity:

We may need to expand all paths, $O(b^m)$

When there are many solutions, DFS may be quicker than BFS

When m is big, much larger than d, ∞ (deep, loops), .. troubles

$\longrightarrow$ Major drawback of DFS: going deep where there is no solution..

Properties:

1. Complete? Not in infinite spaces, complete in finite spaces
2. Optimal?
3. Time? $O(b^m)$ Woow..
   terrible if m is much larger than d, but if solutions are dense, may be much faster than breadth-first
4. Space? $O(bm)$, linear! Woow..

Depth-limited search (I)

⟶ DFS is going too deep, put a threshold on depth!
For instance, 20 cities on map for Romania, any node deeper than 19 is cycling. Don't expand deeper!

⟶ Implement: nodes at depth l have no successor

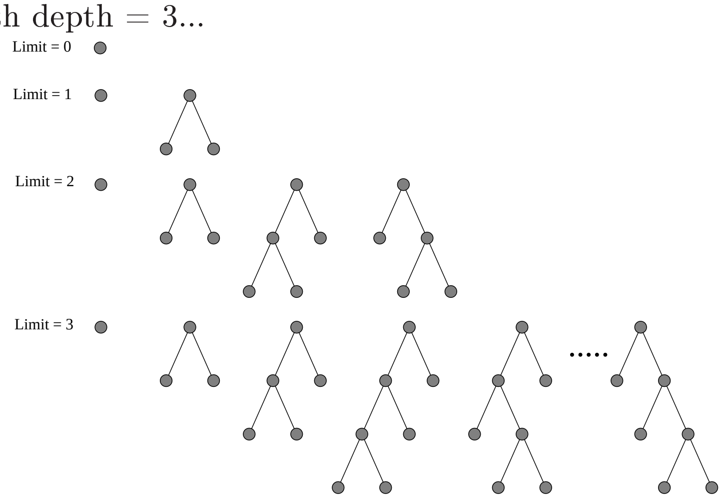
Properties:

1. Complete?
2. Optimal?
3. Time? (given l depth limit)
4. Space? (given l depth limit)

Problem: how to choose l?

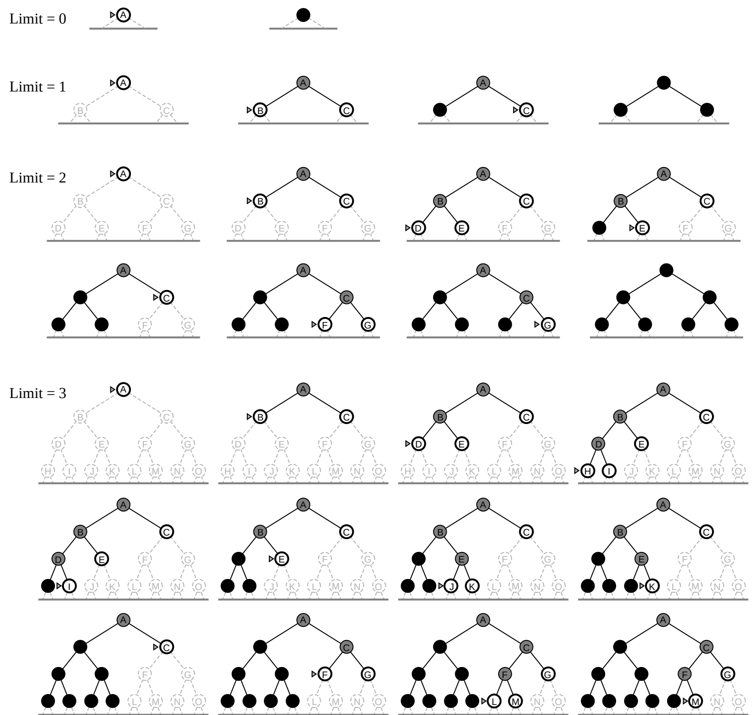
Iterative-deepening search (I)

→ DLS with depth $= 0$
→ DLS with depth $= 1$
→ DLS with depth $= 2$
→ DLS with depth $= 3$...



⟶ Combines benefits of DFS and BFS

Iterative-deepening search (2)



Iterative-deepening search (3)

$\longrightarrow$ combines benefits of DFS and BFS

Properties:

1. Time? $(d+1).b^0 + (d).b + (d-1).b^2 + \ldots + 1.b^d = O(b^d)$
2. Space? $O(bd)$, like DFS
3. Complete? like BFS
4. Optimal? like BFS (if step cost $= 1$)

Iterative-deepening search (4)

$\longrightarrow$ Some nodes are expanded several times, wasteful?

$\text{N(BFS)} = b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - d)$

$\text{N(IDS)} = (d)b + (d-1)b^2 + \ldots + (1)b^d$

Numerical comparison for $b = 10$ and $d = 5$:

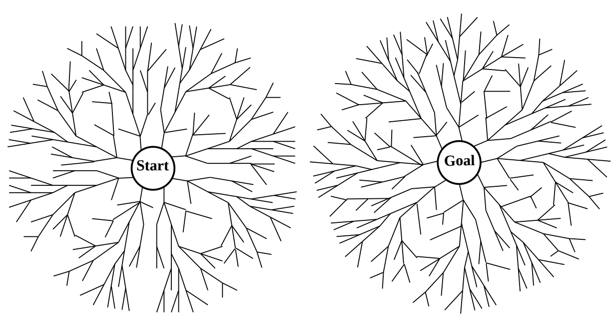
N(IDS) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450

N(BFS) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100

$\longrightarrow$ IDS is preferred when search space is large and depth unknown

Bidirectional search (I)

$\rightarrow$ Given initial state and the goal state, start search from both ends and meet in the middle



$\rightarrow$ Assume same b branching factor, $\exists$ solution at depth d, time: $O(2b^{d/2}) = O(b^{d/2})$

$b = 10, d = 6$, DFS= 1,111,111 nodes, BDS=2,222 nodes!

Bidirectional search (2)

In practice :—(

- Need to define predecessor operators to search backwards
  If operator are invertible, no problem
- What if $\exists$ many goals (set state)?
  do as for multiple-state search
- need to check the 2 fringes to see how they match
  need to check whether any node in one space appears in the other space (use hashing)
  need to keep all nodes in a half in memory $O(b^{d/2})$
- What kind of search in each half space?

Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes*	No	No	Yes

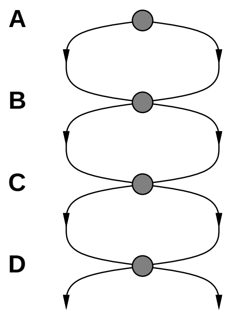
b branching factor
d solution depth
m maximum depth of tree
l depth limit

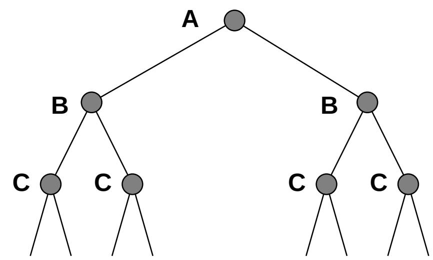
Loops: Avoid repeated states (I)

Avoid expanding states that have already been visited

Valid for both infinite and finite trees

$$\text{Example: } \begin{cases} m \text{ maximum depth} \\ m+1 \text{ states} \\ 2^m \text{ possible branches (paths)} \end{cases}$$



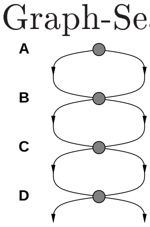


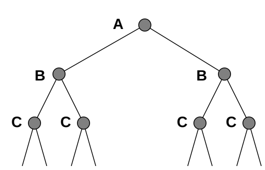
Loops: (2)

$$\text{Keep nodes in two lists: } \begin{cases} \text{Open list: Fringe} \\ \text{Closed list: Leaf and expansed nodes} \end{cases}$$

Discard a current node that matches a node in the closed list

Tree-Search $\longrightarrow$ Graph-Search





Issues:

1. Implementation: hash table, access is constant time
   Trade-off cost of storing+checking vs. cost of searching
2. Losing optimality
   when new path is cheaper/shorter of the one stored
3. DFS and IDS now require exponential storage

Summary

Path: sequence of actions leading from one state to another

Partial solution: a path from an initial state to another state

Search: develop a sets of partial solutions

- Search tree & its components (node, root, leaves, fringe)
- Data structure for a search node
- Search space vs. state space
- Node expansion, queue order
- Search types: uninformed vs. heuristic
- 6 uninformed search strategies
- 4 criteria for evaluating & comparing search strategies