Title: First-Order Logic
AIMA: Chapter 8 (Sections 8.1 and 8.2)
Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence
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Outline

- First-order logic:
  - basic elements
  - syntax
  - semantics
- Examples
**Pros and cons** of propositional logic

- Propositional logic is **declarative**: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
  (unlike most data structures and databases)
- Propositional logic is compositional:
  meaning of $B_{1,1} \land P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent
  (unlike natural language, where meaning depends on context)
- but...
  Propositional logic has very limited expressive power
  E.g., cannot say “pits cause breezes in adjacent squares”
  except by writing one sentence for each square

**Propositional Logic**

- is simple
- illustrates important points:
  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness:
  $\rightarrow$ In PL, world contains **facts**

**First-Order Logic**

- more symbols (objects, properties, relations)
- more connectives (quantifier)
First Order Logic

→ FOL provides more "primitives" to express knowledge:
  — objects (identity & properties)
  — relations among objects (including functions)

**Objects:** people, houses, numbers, Einstein, Huskers, event, ..

**Properties:** smart, nice, large, intelligent, loved, occurred, ..

**Relations:** brother-of, bigger-than, part-of, occurred-after, ..

**Functions:** father-of, best-friend, double-of, ..

**Examples:**
  (objects? function? relation? property?)
  — one plus two equals four  [sic]
  — squares neighboring the wumpus are smelly

Logic

**Attracts:** mathematicians, philosophers and AI people

**Advantages:**
  — allows to represent the world and reason about it
  — expresses anything that can be programmed

**Non-committal to:**
  — symbols could be objects or relations
    (e.g., King(Gustave), King(Sweden, Gustave), Merciless(King))
  — classes, categories, time, events, uncertainty

.. **but amenable** to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

→ Some people think FOL *is* the language of AI
  true/false? donna :—( but it will remain around for some time..
Types of logic

Logics are characterized by what they commit to as “primitives”

**Ontological commitment**: what exists—facts? objects? time? beliefs?

**Epistemological commitment**: what states of knowledge?

<table>
<thead>
<tr>
<th>Language</th>
<th>Ontological Commitment (What exists in the world)</th>
<th>Epistemological Commitment (What an agent believes about facts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>First-order logic</td>
<td>facts, objects, relations</td>
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<tr>
<td>Temporal logic</td>
<td>facts, objects, relations, times</td>
<td>true/false/unknown</td>
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<tr>
<td>Probability theory</td>
<td>facts</td>
<td>degree of belief 0…1</td>
</tr>
<tr>
<td>Fuzzy logic</td>
<td>degree of truth</td>
<td>degree of belief 0…1</td>
</tr>
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</table>

Higher-Order Logic: views relations and functions of FOL as objects

Syntax of FOL: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: $x$, $y$, etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation)
  Father-of, Square-root, LeftLeg, etc.
- Quantifiers $\forall$, $\exists$
- Connectives: $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$
- (Sometimes) equality $=$

Predicates and functions can have any arity (number of arguments)
**Basic elements** in FOL (i.e., the grammar)

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier

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**Term**

Logical expression that refers to an object

- built with: constant symbols, variables, function symbols

\[
\text{Term} = \text{function}(term_1, \ldots, term_n)
\]

or constant or variable

- **ground term**: term with no variable
**Atomic sentences**

state facts

built with terms and predicate symbols

\[
\text{Atomic sentence} = \text{predicate}(\text{term}_1, \ldots, \text{term}_n)
\]

or \( \text{term}_1 = \text{term}_2 \)

**Examples:**

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))

**Complex Sentences**

built with atomic sentences and logical connectives

\( \neg S \)

\( S_1 \land S_2 \)

\( S_1 \lor S_2 \)

\( S_1 \Rightarrow S_2 \)

\( S_1 \Leftrightarrow S_2 \)

**Examples:**

Sibling(KingJohn, Richard) \( \Rightarrow \) Sibling(Richard, KingJohn)

\( (1, 2) \lor \leq (1, 2) \)

\( (1, 2) \land \neg > (1, 2) \)
**Truth in first-order logic:** Semantic

Sentences are true with respect to a model and an interpretation.

Model contains objects and relations among them.

Interpretation specifies referents for:
- constant symbols → objects
- predicate symbols → relations
- function symbols → functional relations

An atomic sentence $\text{predicate}(\text{term}_1, \ldots, \text{term}_n)$ is true iff the objects referred to by $\text{term}_1, \ldots, \text{term}_n$ are in the relation referred to by $\text{predicate}$.

**Model in FOL:** example

The domain of a model is the set of objects it contains:
- five objects

Intended interpretation: Richard refers Richard the Lion Heart, John refers to Evil King John, Brother refers to brotherhood relation, etc.
**Models for FOL:** Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements $n$ from 1 to $\infty$

  For each $k$-ary predicate $P_k$ in the vocabulary
  
    For each possible $k$-ary relation on $n$ objects

  For each constant symbol $C$ in the vocabulary

    For each choice of referent for $C$ from $n$ objects . . .

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded

$\rightarrow$ Checking entailment by enumerating is not an option

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**Quantifiers**

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things
Universal quantification

∀ (variables) (sentence)

Example: all dogs like bones ∀ x Dog(x) ⇒ LikeBones(x)
x = Indy is a dog x = Indiana Jones is a person

∀ x P is equivalent to the conjunction of instantiations of P

Dog(Indy) ⇒ LikeBones(Indy)
∧ Dog(Rebel) ⇒ LikeBones(Rebel)
∧ Dog(KingJohn) ⇒ LikeBones(KingJohn)
∧ ...

Typically: ⇒ is the main connective with ∀

Common mistake: using ∧ as the main connective with ∀

Example: ∀ x Dog(x) ∧ LikeBones(x)
all objects in the world are dogs, and all like bones

Existential quantification

∃ (variables) (sentence)

Example: some student will talk at the TechFair
∃ x Student(x) ∧ TalksAtTechFair(x)
Pat, Leslie, Chris are students
∃ x P is equivalent to the disjunction of instantiations of P

Student(Pat) ∧ TalksAtTechFair(Pat)
∨ Student(Leslie) ∧ TalksAtTechFair(Leslie)
∨ Student(Chris) ∧ TalksAtTechFair(Chris)
∨ ...

Typically: ∧ is the main connective with ∃

Common mistake: using ⇒ as the main connective with ∃

∃ x Student(x) ⇒ TalksAtTechFair(x)
is true if there is anyone who is not Student
Properties of quantifiers (I)

∀x ∀y is the same as ∀y ∀x
∃x ∃y is the same as ∃y ∃x
∃x ∀y is not the same as ∀y ∃x
∃x ∀y Loves(x, y)
“There is a person who loves everyone in the world”
∀y ∃x Loves(x, y)
“Everyone in the world is loved by at least one person”

Quantifier duality: each can be expressed using the other
∀x Likes(x, IceCream)    ¬∃x ¬Likes(x, IceCream)
∃x Likes(x, Broccoli)    ¬∀x ¬Likes(x, Broccoli)

Parsimony principal: ∀, ¬, and ⇒ are sufficient

Properties of quantifiers (II)

Nested quantifier:
∀ x(∃ y(P(x, y))):
every object in the world has a particular property, which is the property to be related to some object by the relation P
∃ x (∀ y(P(x, y))):
there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

Lexical scoping: ∀ x[Cat(x) ∨ ∃ x Brother(Richard, x)]

Well-formed formulas (WFF):
(kind of correct spelling)
every variable must be introduced by a quantifier
∀ x P(y) is not a WFF
Examples

Brothers are siblings

“Sibling” is symmetric

One’s mother is one’s female parent

A first cousin is a child of a parent’s sibling

Examples

∀x, y Brother(x, y) ⇒ Sibling(x, y)

∀x, y Sibling(x, y) ⇒ Sibling(y, x)

∀x, y Mother(x, y) ⇒ (Female(x) ∧ Parent(x, y))

∀x, y FirstCousin(x, y) ⇔ ∃a, b Parent(a, x) ∧ Sibling(a, b) ∧ Parent(b, y)
**Tricky example**

Someone is loved by everyone
\[ \exists x \ \forall y \ \text{Loves}(y, x) \]

Someone with red-hair is loved by everyone
\[ \exists x \ \forall y \ \text{Redhair}(x) \land \text{Loves}(y, x) \]

Alternatively:
\[ \exists x \ \text{Person}(x) \land \text{Redhair}(x) \land (\forall y \ \text{Person}(y) \Rightarrow \text{Loves}(y, x)) \]

**Equality**

\( \text{term}_1 = \text{term}_2 \) is true under a given interpretation if and only if \( \text{term}_1 \) and \( \text{term}_2 \) refer to the same object.

Examples

- Father(John) = Henry
- \( 1 = 2 \) is satisfiable
- \( 2 = 2 \) is valid
- Useful to distinguish two objects:
  - Definition of (full) Sibling in terms of Parent:
    \[ \forall x, y \ \text{Sibling}(x, y) \iff \neg(x = y) \land \exists m, f \neg(m = f) \land \text{Parent}(m, x) \land \text{Parent}(f, x) \land \text{Parent}(m, y) \land \text{Parent}(f, y) \]
  - Spot has at least two sisters: ...

AIMA, Exercise 8.4 & 8.7
Knowledge representation (KR)

**Domain:** a section of the world about which we wish to express some knowledge

**Example:** Family relations (kinship):
- Objects: people
- Properties: gender, married, divorced, single, widowed
- Relations: parenthood, brotherhood, marriage...

**Unary predicates:** Male, Female

**Binary relations:** Parent, Sibling, Brother, Child, etc.

**Functions:** Mother, Father

\[ \forall m, c, \text{Mother}(c) = m \iff \text{Female}(m) \land \text{Parent}(m, c) \]

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**In Logic** (informally)

- Basic facts: **axioms**
- Derived facts: **theorems**

**Independent axiom**

an axiom that cannot be derived from the rest

\[ \rightarrow \text{Goal of mathematicians: find the minimal set} \]

**In AI**

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK
Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t = 5$:

$\textit{Tell}(KB, \textit{Percept}([\textit{Smell, Breeze, None}], 5))$

$\textit{Ask}(KB, \exists a \textit{Action}(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: Yes, $\{a/\text{Shoot}\}$ ← substitution (binding list)

Given a sentence $S$ and a substitution $\sigma$,

$S\sigma$ denotes the result of plugging $\sigma$ into $S$; e.g.,

$S = \textit{Smarter}(x, y)$

$\sigma = \{x/\text{Hillary, y/Bill}\}$

$S\sigma = \textit{Smarter}(\text{Hillary, Bill})$

$\textit{Ask}(KB, S)$ returns some/all $\sigma$ such that $KB \models S\sigma$

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Prepare for next lecture: AIMA, Exercise 8.24, page 319

Takes($x, c, s$): student $x$ takes course $c$ in semester $s$

Passes($x, c, s$): student $x$ passes course $c$ in semester $s$

Score($x, c, s$): the score obtained by student $x$ in course $c$ in semester $s$

$x > y$: $x$ is greater than $y$

$F$ and $G$: specific French and Greek courses

Buys($x, y, z$): $x$ buys $y$ from $z$

Sells($x, y, z$): $x$ sells $y$ from $z$

Shaves($x, y$): person $x$ shaves person $y$

Born($x, c$): person $x$ is born in country $c$

Parent($x, y$): person $x$ is parent of person $y$

Citizen($x, c, r$): person $x$ is citizen of country $c$ for reason $r$

Resident($x, c$): person $x$ is resident of country $c$ of person $y$

Fools($x, y, t$): person $x$ fools person $y$ at time $t$

Student($x$), Person($x$), Man($x$), Barber($x$), Expensive($x$), Agent($x$), Insured($x$), Smart($x$), Politician($x$),
AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986
(then: University of Rochester
then: head of AI at AT&T Labs-Research
and Program co-chair of AAAI-2000

Now: Associate Professor at University of Washington, Seattle)