

Title: Adversarial Search
AIMA: Chapter 5 (Sections 5.1, 5.2 and 5.3)

Introduction to Artificial Intelligence
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Outline

- Introduction
- Minimax algorithm
- Alpha-beta pruning

Context

- In an MAS, agents affect each other's welfare
- Environment can be cooperative or competitive
- Competitive environments yield adversarial search problems (games)
- Approaches: mathematical game theory and AI games

Game theory vs. AI

- AI games: fully observable, deterministic environments, players alternate, utility values are equal (draw) or opposite (winner/loser)

In vocabulary of game theory: deterministic, turn-taking, two-player, zero-sum games of perfect information

- Games are attractive to AI: states simple to represent, agents restricted to a small number of actions, outcome defined by simple rules

Not croquet or ice hockey, but typically board games

Exception: Soccer (Robocup www.robotcup.org/)

Board game playing: an appealing target of AI research

Board game: Chess (since early AI), Othello, Go, Backgammon, etc.

- Easy to represent
- Fairly small numbers of well-defined actions
- Environment fairly accessible
- Good abstraction of an enemy, w/o real-life (or war) risks :—)

But also: Bridge, ping-pong, etc.

Characteristics

- 'Unpredictable' opponent: contingency problem (interleaves search and execution)
- Not the usual type of 'uncertainty':
no randomness/no missing information (such as in traffic)
but, the moves of the opponent expectedly non benign
- Challenges:
 - huge branching factor
 - large solution space
 - Computing optimal solution is infeasible
 - Yet, decisions must be made. Forget A*...

Discussion

- What are the theoretically best moves?
- Techniques for choosing a good move when time is tight
 - ✓ Pruning: ignore irrelevant portions of the search space
 - × Evaluation function: approximate the true utility of a state without doing search

Two-person Games

- 2 player: Min and Max
- Max moves first
- Players alternate until end of game
- Gain awarded to player/penalty give to loser

Game as a search problem:

- Initial state: board position & indication whose turn it is
- Successor function: defining legal moves a player can take
Returns $\{(move, state)^*\}$
- Terminal test: determining when game is over
states satisfy the test: terminal states
- Utility function (a.k.a. payoff function): numerical value for outcome e.g., Chess: win=1, loss=-1, draw=0

Usual search

Max finds a sequence of operators yielding a terminal goal scoring winner according to the utility function

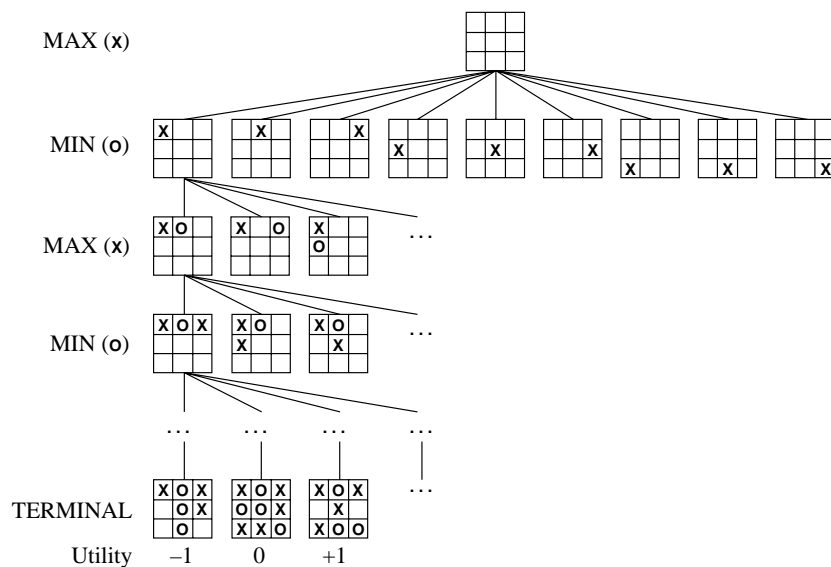
Game search

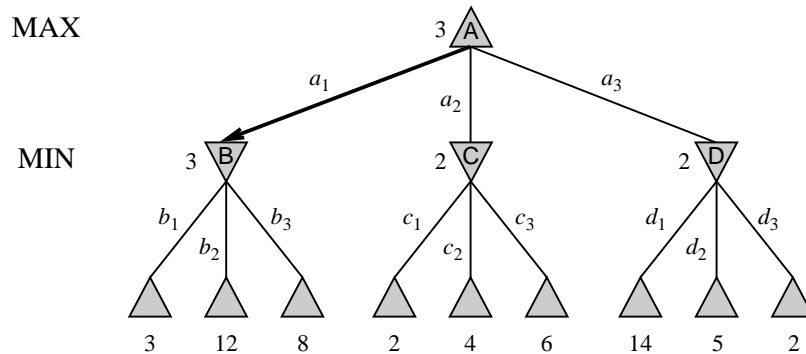
- Min actions are significant
Max must find a strategy to win regardless of what Min does:
→ correct action for Max for each action of Min
- Need to approximate (no time to envisage all possibilities difficulty): a huge state space, an even more huge search space
e.g., chess: $\left\{ \begin{array}{l} 10^{40} \text{ different legal positions} \\ \text{Average branching factor}=35, 50 \text{ moves/player}= 35^{100} \end{array} \right.$
- Performance in terms of time is very important

Example: Tic-Tac-Toe

Max has 9 alternative moves

Terminal states' utility: Max wins=1, Max loses = -1, Draw = 0



Example: 2-ply game treeMax's actions: a_1, a_2, a_3 Min's actions: b_1, b_2, b_3 

Minimax algorithm determines the optimal strategy for Max
 → decides which is the best move

Minimax algorithm

- Generate the whole tree, down to the leaves
- Compute utility of each terminal state
- Iteratively, from the leaves up to the root, use utility of nodes at depth d to compute utility of nodes at depth $(d - 1)$:

MIN 'row': minimum of children

MAX 'row': maximum of children

MINIMAX-VALUE (n)

$$\begin{cases} \text{UTILITY}(n) & \text{if } n \text{ is a terminal node} \\ \max_{s \in \text{Succ}(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a Max node} \\ \min_{s \in \text{Succ}(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a Min node} \end{cases}$$

Minimax decision

- MAX's decision: minimax decision maximizes utility under the assumption that the opponent will play perfectly to his/her own advantage
- Minimax decision maximizes the worst-case outcome for Max (which otherwise is guaranteed to do better)
- If opponent is sub-optimal, other strategies may reach better outcome better than the minimax decision

Minimax algorithm: Properties

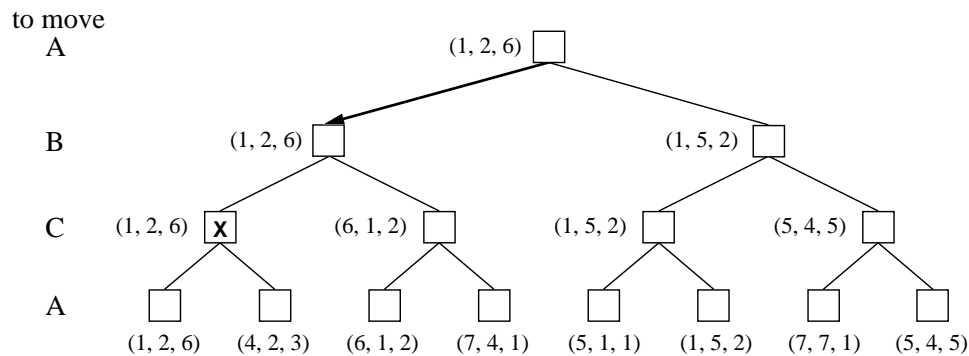
- m maximum depth
 b legal moves
- Using Depth-first search, space requirement is:
 $O(bm)$: if generating all successors at once
 $O(m)$: if considering successors one at a time
- Time complexity $O(b^m)$
Real games: time cost totally unacceptable

Multiple players games

UTILITY(n) becomes a vector of the size of the number of players

For each node, the vector gives the utility of the state for each player

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Alliance formation in multiple players games

How about alliances?

- A and B in weak positions, but C in strong position
A and B make an alliance to attack C (rather than each other
→ Collaboration emerges from purely selfish behavior!
- Alliances can be done and undone (careful for social stigma!)
- When a two-player game is not zero-sum, players may end up automatically making alliances (for example when the terminal state maximizes utility of both players)

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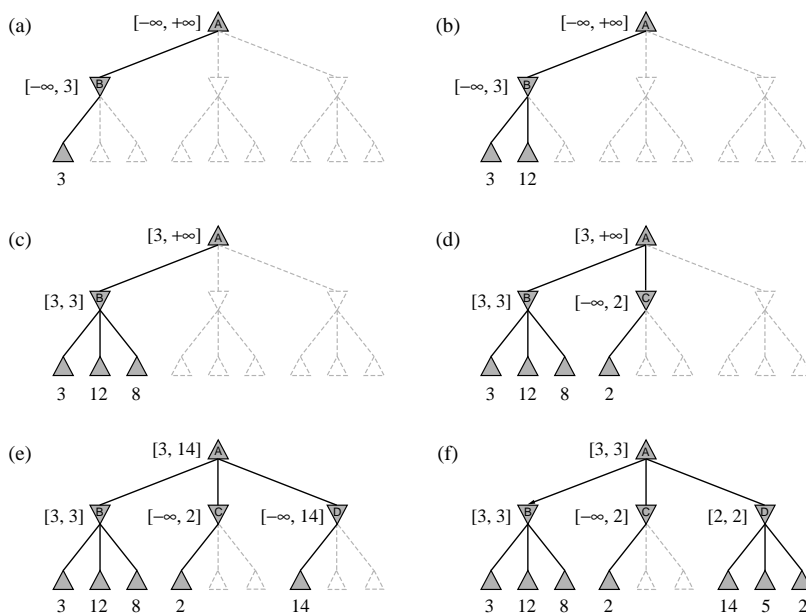
Alpha-beta pruning

- Minimax requires computing all terminal nodes: unacceptable
- Do we really need to do compute utility of all terminal nodes?
... No, says John McCarthy in 1956:

It is possible to compute the correct minimax decision without looking at every node in the tree, and yet get the correct decision

- Use pruning (eliminating useless branches in a tree)

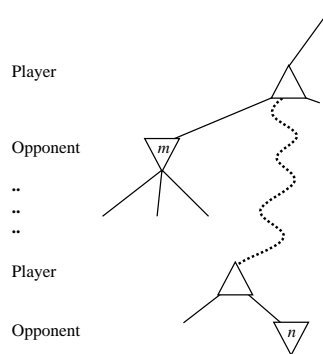
Example of alpha-beta pruning



Try 14, 5, 2, 6 below D

General principal of Alpha-beta pruning

If Player has a better choice m at $\left\{ \begin{array}{l} \text{— a parent node of } n \\ \text{— any choice point further up} \end{array} \right.$
 n will never be reached in actual play

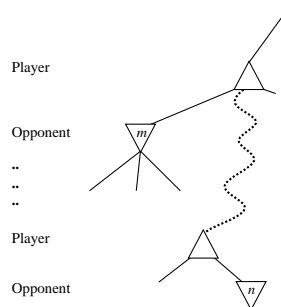


Once we have found enough about n (e.g., through one of its descendants), we can prune it (i.e., discard all its remaining descendants)

Mechanism of Alpha-beta pruning

α : value of best choice so far for MAX, (maximum)

β : value of best choice so far for MIN, (minimum)

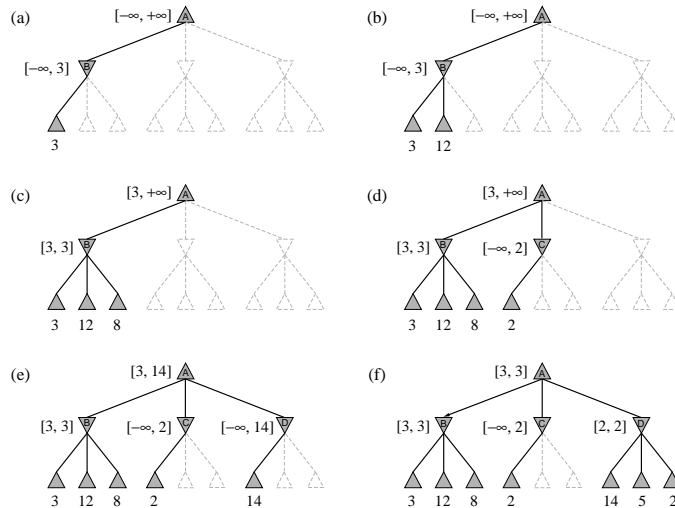


Alpha-beta search:

- updates the value of α , β as it goes along
- prunes a subtree as soon as its worse then current α or β

Effectiveness of pruning

Effectiveness of pruning depends on the order of new nodes examined



Savings in terms of cost

- Ideal case:
Alpha-beta examines $O(b^{d/2})$ nodes (vs. Minimax: $O(b^d)$)
→ Effective branching factor \sqrt{b} (vs. Minimax: b)
- Successors ordered randomly:
 $b > 1000$, asymptotic complexity is $O((b/\log b)^d)$
 b reasonable, asymptotic complexity is $O(b^{3d/4})$
- Practically: Fairly simple heuristics work (fairly) well