Homework 7

Due: Friday, April 5, 2010.

You can choose to do either Section 1 (worth 100 points) or Sections 2 to 9 (with 100 points plus 20 bonus points). It is your decision. If you choose to do both options, you will receive the maximum of the grades of the two sections, not the sum.

1 Implementation: Solving SAT 100 points

Write a search algorithm to determine the satisfiability of a SAT instance. You can either write:

- A DPLL procedure (backtrack search),
- A local search procedure.

You must

- Clearly describe, in addition to your code, your data structures, how your search algorithm operates, and the improvements, if any, that you have included in your code.
- We recommend that you use the standard file for input files known as the ‘simplified version of the DIMACS format’: http://www.satcompetition.org/2009/format-benchmarks2009.html
- Test the performance of your algorithm on some non trivial uniform random instances taken from the SAT Competition. For example: http://www.cs.ubc.ca/~hoos/SATLIB/benchm.html

Alert: many implementations exist in the literature and on the web. We expect you to do your own implementation.

2 AIMA, Exercise 7.1, page 279. 16 points

3 AIMA, Exercise 7.7, page 281. 6 points

4 Truth Tables 8 points

Use truth tables to show that each of the following is a tautology.
1. \((p \land q) \to \neg(\neg p \lor \neg q)\)
2. \([Mary \land (Mary \to Susy)] \to Susy\)
3. \(\alpha \to [\beta \to (\alpha \land \beta)]\)
4. \((a \to b) \to [(b \to c) \to (a \to c)]\)

5  AIMA, Exercise 7.10, page 281.  16 points

Only b, c, d, e, f, and g.

6  Logical Equivalences  8 points

Using a method of your choice, verify:

1. \((\alpha \to \beta) \equiv (\neg \beta \to \neg \alpha)\) contraposition
2. \(\neg(\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta)\) de Morgan
3. \((\alpha \land (\beta \lor \gamma)) \equiv ((\alpha \lor \gamma) \lor (\alpha \land \beta))\) distributivity of \(\land\) over \(\lor\)

7  AIMA, Exercise 7.22, page 284.  18 points + 20 bonus

Parts a, b, and c are required. Parts d, e, and f are bonus.

8  Proofs  28 points

Give the explanations of each step if the steps are given, and give both the explanation and step if they are not.

- If \(q \land (r \land p), t \to v, v \to \neg p,\) then \(\neg t \land r.\)

  Proof  
  
  1. \(q \land (r \land p)\)  Given
  2. \(t \to v\)  Given
  3. \(v \to \neg p\)  Given
  4. \(t \to \neg p\)  Given
  5. \((r \land p)\)  Given
  6. \(r\)  Given
  7. \(p\)  Given

Explanations
8. \( \neg \neg p \)
9. \( \neg t \)
10. \( \neg t \land r \)

- If \( p \rightarrow (q \land r), q \rightarrow s, \) and \( r \rightarrow t, \) then \( p \rightarrow (s \land t). \)

Proof

1. 
2. 
3. 
4. 
5. 
6. 
7.

Explanations

- Prove by contradiction.

If \( \neg (\neg p \land q), p \rightarrow (\neg t \lor r), q, \) and \( t, \) then \( r. \)

Proof

1. \( \neg (\neg p \land q) \)  
2. \( p \rightarrow (\neg t \lor r) \)  
3. \( q \)  
4. \( t \)  
5. \( \neg r \)  
6. 
7. 
8. 
9. 
10. 
11. 
12.

Explanations

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \neg (\neg p \land q) )</td>
<td>Given</td>
</tr>
<tr>
<td>2</td>
<td>( p \rightarrow (\neg t \lor r) )</td>
<td>Given</td>
</tr>
<tr>
<td>3</td>
<td>( q )</td>
<td>Given</td>
</tr>
<tr>
<td>4</td>
<td>( t )</td>
<td>Given</td>
</tr>
<tr>
<td>5</td>
<td>( \neg r )</td>
<td>Negation of Conclusion</td>
</tr>
</tbody>
</table>