**Problem A** Let

\[ f(n) = (k_1)^{c_1 n} \]
\[ g(n) = (k_2)^{c_2 n} \]

Where \( k_1, k_2, c_1, c_2 \) are all real numbers greater than 1. Under what conditions can you say that \( f(n) \in \mathcal{O}(g(n)) \)?

**Problem B** Prove that

\[ 1^k + 2^k + \cdots + n^k \in \mathcal{O}(n^{k+1}) \]
Problem C Order the following functions in non-decreasing order of growth. You need not give a formal proof for each.

1. \(6n \log(n) + 2n, \quad \left(\frac{1}{3}\right)^n, \quad n^n, \quad \log \log(n).\)
2. \(\log^2(n), \quad \frac{1}{n}, \quad 2^{10}, \quad n - n^3 + 6n^5, \quad \frac{n}{\log(n)}, \quad n!\)
3. \(2^{\log(n)}, \quad 2^n, \quad 2^4n, \quad 4^2n, \quad 3n + \log(n^{100}), \quad \log(n) \log \log(n)\)

Problem D Give and analyze an algorithm for the following problem. Write clearly and neatly your pseudocode. \LaTeX{} users can use algorithm2e.sty package, which can be easily found on the web.

Given an \(n\) vertex convex polygon described by coordinates \(\{(x_1, y_1), \ldots, (x_n, y_n)\}\), find the three vertices whose corresponding triangle has maximum perimeter.

Problem E Give a tight bound of the form \(f(n) \in \Delta(g(n))\) for the following pairs of functions, knowing that \((\log_b(f(x)))' = \frac{f'(x)}{f(x) \ln b}\):

1. \(f(x) = x^2 \log x\) and \(g(x) = x^3\)
2. \(f(x) = x^4 + \log(3x^8 + 7)\) and \(g(x) = (x^2 + 17x + 3)^2\)
3. \(f(x) = \log(x^2 + 1)\) and \(g(x) = \log(x)\).
4. \(f(x) = 2^{2^x}\) and \(g(x) = 2^x\)

Problem F Suppose that \(f(x), g(x)\) and \(h(x)\) are positive functions such that \(f(x) \in O(g(x))\) and \(g(x) \in O(h(x))\). Show that \(f(x) \in O(h(x))\). Write your proof as formally and neatly as possible.

Instructions Follow instructions carefully, failure to do so may result in points being deducted.

- The homework can be submitted on paper or via handin. Homework neatly formatted in \LaTeX{} will receive an 8 point bonus. You will not receive the 8 points bonus if you work with a partner (see below).
- Clearly label each problem and submit the answers in order.
- Staple this cover page to the front of your assignment for easier grading.
- Late submissions will not be accepted.
- Show sufficient work to justify your answer(s). Sloppy, hard to read papers will not be graded (please have mercy...).
- When you are asked to prove something, you must give as formal, rigorous, and complete a proof as possible.
- You are to work individually, and all work should be your own. Check partner policy below.
- The CSE academic dishonesty policy is in effect (see \url{http://www.cse.unl.edu/undergrads/academic_integrity.php}).

Partner Policy You may work in pairs, but you must follow these guidelines:

1. You must work on all problems together. You may not simply partition the work between you.
2. You must use \LaTeX{} and you may divide the typing duties however you wish.
3. You may not discuss problems with other groups or individuals.
4. Hand in only one hard copy under the first author’s name.