

## Bonus Homework 9

**Assigned on:** Friday April 17, 2009.

**Due:** Friday April 24, 2009.

This is a pen-and-paper homework, to be returned in class or with web handin.  
The whole homework is worth 105 points (all bonus points).

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## 1 Unification (4 points)

What is the most general unifier of the following pairs of wff's? If none exists, report "fail."  
Assume that the capital letters are constants and the lowercase letters are variables.

1.  $P(x, y, x, z)$  and  $P(F(w), A, F(B), w)$
2.  $Q(x, F(x), G(F(x)))$  and  $Q(1, y, G(F(y)))$
3.  $\text{Foo}(x, y)$  and  $\text{Foo}(y, x)$
4.  $\text{Mother}(x, y)$  and  $\text{Mother}(y, \text{Father}(x))$

## 2 Unification and Resolution (2 points)

You are given the following pairs of clauses where upper case letters indicate constants and lower case letters indicate variables, functions, or predicates. Consider each pair independently of the others. In each pair, variables with the same name are meant to be the same variable. For each of the pairs, specify if the two clauses can be resolved. If yes, show the results of the unification process. If not, explain why.

1.  $p(B, C, x, z, f(A, z, B))$  and  $\neg p(y, z, y, C, w)$ .
2.  $r(f(y), y, x)$  and  $\neg r(x, f(A), f(v))$ .

## 3 Deduction (5 points)

Using the inference rules for logic prove that “ $\exists xZ(x)$ ” follows from the givens. Alert: You will need to use the rule called *instantiation generalization* which we did not cover in class but we did cover in CSE235<sup>1</sup>. Be sure to justify your steps by stating the inference rule used, along with the previous line(s) to which it was applied and the unifications used.

- |    |  |       |
|----|--|-------|
| 1. | $P(1)$   | given |
| 2. | $W(1) \wedge W(2) \wedge W(3)$                   | given |
| 3. | $\forall x[P(x) \Rightarrow \neg R(x)]$          | given |
| 4. | $\forall x[Q(x) \vee R(x)]$                      | given |
| 5. | $\forall x[(Q(x) \wedge W(x)) \Rightarrow Z(x)]$ | given |

## 4 Resolution and Refutation (5 points)

Use resolution and refutation to solve the problem below. Hint: First transform the givens into clausal form.

Given:

1.  $\forall x(P(x) \Rightarrow Q(x))$
2.  $\forall x(P(x) \Rightarrow (\exists yW(y)))$
3.  $\forall x\forall y((Q(x) \wedge W(y)) \Rightarrow s(x))$
4.  $P(Mary)$

Show:  $S(Mary)$

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<sup>1</sup>Search the web or check your textbook CSE235 textbook, which is available from the CSE Permanent Reserves at the Math Library in Avery Hall.

## 5 Translation into FOL (14 points)

Consider the set of all creatures. We will use the following predicates:

- $Insect(x)$ .
- $Moth(x)$ .
- $Dragonfly(x)$ .
- $Spider(x)$ .
- $Eats(x, y)$ .
- $Wings(x, y)$ . ( $x$  has  $y$  wings.)
- $Order(x)$ . (Recall that zoologists classify creatures using kingdom, phylum, class, order, family, genus, and species.)

Translate the following sentences into logic. You may only use the identity predicate and the predicates and functions listed above. Try not to include more bugs in your logic than are required..

- Flik is an insect with 4 wings but is not a moth.
- Not all insects have 4 wings.
- All insects with 2 wings are in the same order.
- There are at least 3 difference orders.
- Moths and Dragonflies are insects but are not in the same order.
- All spiders eat insects.
- Some spiders eat only insects.

## 6 Inference in First-Order Logic: CNF and Resolution (75 points)

For *each* of the exercises below, answer the following:

- Choose appropriate predicates to write the above axioms in first-order logic.
- Write the axioms in First-Order Logic. Report your results neatly.
- Transform each of the first-order sentences into Conjunctive Normal Form. Clearly state the Skolem functions and clearly number the statements. Neatly report your results and provide as much detail as possible.
- Establish the conclusion using the axioms by applying *refutation resolution*. That is, negate the conclusion and prove the unsatisfiability of the set of clauses by resolution. Clearly show the variable bindings at each step and clearly number the statements.

### **6.1 Do Loons Eat Fish? (15 points)**

Consider the following axioms.

1. Every bird sleeps in some tree.
2. Every loon is a bird, and every loon is aquatic.
3. Every tree in which any aquatic bird sleeps is beside some lake.
4. Anything that sleeps in anything that is beside any lake eats fish.
5. (Conclusion) Every loon eats fish.

### **6.2 Is there a conservative Austinite? (15 points)**

Consider the following axioms:

1. Every Austinite who is not conservative loves same armadillo.
2. Anyone who wears maroon-and-white shirts is an Aggie.
3. Every Aggie loves every dog.
4. Nobody who loves every dog loves any armadillo.
5. Clem is an Austinite, and Clem wears maroon-and-white shirts.
6. (Conclusion) Is there a conservative Austinite?

### **6.3 Will Mary Date John? (15 points)**

Consider the following axioms:

1. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
2. Every dog chases some rabbit.
3. Mary buys carrots by the bushel.
4. Anyone who owns a rabbit hates anything that chases any rabbit.
5. John owns a dog.
6. Someone who hates something owned by another person will not date that person.
7. (Conclusion) If Mary does not own a grocery store, she will not date John.

## 6.4 Drunk AI Students (15 points)

Consider the following axioms.

1. Everyone who feels warm either is drunk, or every costume they have is warm.
2. Every costume that is warm is furry.
3. Every AI student is a CS student.
4. Every AI student has some robot costume.
5. No robot costume is furry.
6. (Conclusion) If every CS student feels warm, then every AI student is drunk.

## 6.5 Brilliant CS Students (15 points)

Consider the following axioms:

1. Every student who makes good grades is brilliant or studies.
2. Every student who is a CS major has some roommate. [Make “roommate” a two-place predicate.]
3. Every student who has any roommate who likes to party goes to Sixth Street.
4. Anyone who goes to Sixth Street does not study.
5. (Conclusion) If every roommate of every CS major likes to party, then every student who is a CS major and makes good grades is brilliant.