Title: First-Order Logic
AIMA: Chapter 8 (Sections 8.1 and 8.2)

Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence
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Outline

• First-order logic:
  – basic elements
  – syntax
  – semantics

• Examples
Pros and cons of propositional logic

- Propositional logic is **declarative**: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases)
- Propositional logic is compositional: meaning of $B_{1,1} \land P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent (unlike natural language, where meaning depends on context)
- but...

  Propositional logic has very limited expressive power
  E.g., cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square

Propositional Logic

- is simple
- illustrates important points:
  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness:
  $\rightarrow$ In PL, world contains **facts**

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)
First Order Logic

→ FOL provides more "primitives" to express knowledge:
  — objects (identity & properties)
  — relations among objects (including functions)

**Objects:** people, houses, numbers, Einstein, Huskers, event, ..
**Properties:** smart, nice, large, intelligent, loved, occurred, ..
**Relations:** brother-of, bigger-than, part-of, occurred-after, ..
**Functions:** father-of, best-friend, double-of, ..

**Examples:** (objects? function? relation? property?)
  — one plus two equals four [sic]
  — squares neighboring the wumpus are smelly

Logic

**Attracts:** mathematicians, philosophers and AI people

**Advantages:**
  — allows to represent the world and reason about it
  — expresses anything that can be programmed

**Non-committal to:**
  — symbols could be objects or relations
    *(e.g., King(Gustave), King(Sweden, Gustave), Merciless(King))*
  — classes, categories, time, events, uncertainty

**but amenable to** extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

→ Some people think FOL *is* the language of AI
  true/false? *donno* :—( but it will remain around for some time..
**Types of logic**

Logics are characterized by what they commit to as “primitives”

**Ontological commitment**: what exists—facts? objects? time? beliefs?

**Epistemological commitment**: what states of knowledge?

<table>
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<tr>
<th>Language</th>
<th>Ontological Commitment (What exists in the world)</th>
<th>Epistemological Commitment (What an agent believes about facts)</th>
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<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
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<td>First-order logic</td>
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<td>Temporal logic</td>
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<td>Probability theory</td>
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Higher-Order Logic: views relations and functions of FOL as objects

**Syntax of FOL**: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: $x$, $y$, etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation) Father-of, Square-root, LeftLeg, etc.
- Quantifiers $\forall$, $\exists$
- Connectives: $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$
- (Sometimes) equality $=$

Predicates and functions can have any arity (number of arguments)
Basic elements in FOL (i.e., the grammar)

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier

Term

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

Term \(= \text{function}(\text{term}_1, \ldots, \text{term}_n)\)

or constant or variable

— ground term: term with no variable
**Atomic sentences**

state facts

built with terms and predicate symbols

\[
\text{Atomic sentence} = \text{predicate}(\text{term}_1, \ldots, \text{term}_n)
\]

or \(\text{term}_1 = \text{term}_2\)

**Examples:**

Brother (Richard, John)
Married (FatherOf(Richard), MotherOf(John))

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**Complex Sentences**

built with atomic sentences and logical connectives

\(-S\)

\(S_1 \land S_2\)

\(S_1 \lor S_2\)

\(S_1 \Rightarrow S_2\)

\(S_1 \Leftrightarrow S_2\)

**Examples:**

Sibling(King,John,Richard) \(\Rightarrow\) Sibling(Richard,King,John)

\((1, 2) \lor \leq (1, 2)\)

\((1, 2) \land \neg \geq (1, 2)\)
**Truth in first-order logic**: Semantic

Sentences are true with respect to a **model** and an **interpretation**

Model contains objects and relations among them

Interpretation specifies referents for

- **constant symbols** → objects
- **predicate symbols** → relations
- **function symbols** → functional relations

An atomic sentence $\text{predicate}(term_1, \ldots, term_n)$ is true iff the objects referred to by $term_1, \ldots, term_n$ are in the relation referred to by $\text{predicate}$

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**Model in FOL**: example

The **domain** of a model is the set of objects it contains:

- five objects

Intended interpretation: Richard refers Richard the Lion Heart, John refers to Evil King John, Brother refers to brotherhood relation, etc.
**Models for FOL:** Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements $n$ from 1 to $\infty$

For each $k$-ary predicate $P_k$ in the vocabulary

For each possible $k$-ary relation on $n$ objects

For each constant symbol $C$ in the vocabulary

For each choice of referent for $C$ from $n$ objects . . .

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded

$\rightarrow$ Checking entailment by enumerating is not an option

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**Quantifiers**

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things

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**Universal quantification**

\( \forall \) (variables) (sentence)

**Example:** all dogs like bones \( \forall \ x \)Dog\( (x) \Rightarrow \)LikeBones\( (x) \)
\( x = \) Indy is a dog \( x = \) Indiana Jones is a person

\( \forall \ x \ P \) is equivalent to the conjunction of instantiations of \( P \)

\[ \text{Dog}(\text{Indy}) \Rightarrow \text{LikeBones}(\text{Indy}) \]
\[ \land \text{Dog}(\text{Rebel}) \Rightarrow \text{LikeBones}(\text{Rebel}) \]
\[ \land \text{Dog}(\text{KingJohn}) \Rightarrow \text{LikeBones}(\text{KingJohn}) \]
\[ \land \ldots \]

**Typically:** \( \Rightarrow \) is the main connective with \( \forall \)

**Common mistake:** using \( \land \) as the main connective with \( \forall \)

Example: \( \forall \ x \) Dog\( (x) \land \)LikeBones\( (x) \)

all objects in the world are dogs, and all like bones

**Existential quantification**

\( \exists \) (variables) (sentence)

**Example:** some student will talk at the TechFair

\( \exists \ x \)Student\( (x) \land \)TalksAtTechFair\( (x) \)

Pat, Leslie, Chris are students

\( \exists \ x \ P \) is equivalent to the disjunction of instantiations of \( P \)

\[ \text{Student}(\text{Pat}) \land \text{TalksAtTechFair}(\text{Pat}) \]
\[ \lor \text{Student}(\text{Leslie}) \land \text{TalksAtTechFair}(\text{Leslie}) \]
\[ \lor \text{Student}(\text{Chris}) \land \text{TalksAtTechFair}(\text{Chris}) \]
\[ \lor \ldots \]

**Typically:** \( \lor \) is the main connective with \( \exists \)

**Common mistake:** using \( \Rightarrow \) as the main connective with \( \exists \)

\( \exists \ x \) Student\( (x) \Rightarrow \)TalksAtTechFair\( (x) \)

is true if there is anyone who is not Student
Properties of quantifiers (I)

∀x ∀y is the same as ∀y ∀x
∃x ∃y is the same as ∃y ∃x
∃x ∀y is not the same as ∀y ∃x
∃x ∀y Loves(x, y)
“There is a person who loves everyone in the world”
∀y ∃x Loves(x, y)
“Everyone in the world is loved by at least one person”

Quantifier duality: each can be expressed using the other
∀x Likes(x, IceCream) ⊃ ¬∃x ¬Likes(x, IceCream)
∃x Likes(x, Broccoli) ⊃ ¬∀x ¬Likes(x, Broccoli)

Parsimony principal: ∀, ¬, and ⇒ are sufficient

Properties of quantifiers (II)

Nested quantifier:
∀x(∃y(P(x, y)):
every object in the world has a particular property, which is the property to be related to some object by the relation P
∃x (∀y(P(x, y)):
there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

Lexical scoping: ∀x[Cat(x) ∨ ∃xBrother(Richard, x)]

Well-formed formulas (WFF): (kind of correct spelling)
every variable must be introduced by a quantifier
∀xP(y) is not a WFF
Examples

Brothers are siblings

“Sibling” is symmetric

One’s mother is one’s female parent

A first cousin is a child of a parent’s sibling

∀x, y Brother(x, y) ⇒ Sibling(x, y)

∀x, y Sibling(x, y) ⇒ Sibling(y, x)

∀x, y Mother(x, y) ⇒ (Female(x) ∧ Parent(x, y))

∀x, y FirstCousin(x, y) ⇔ ∃a, b Parent(a, x) ∧ Sibling(a, b) ∧ Parent(b, y)
Tricky example

Someone is loved by everyone
\[ \exists x \forall y \text{Loves}(y, x) \]

Someone with red-hair is loved by everyone
\[ \exists x \forall y \text{Redhair}(x) \land \text{Loves}(y, x) \]

Alternatively:
\[ \exists x \text{Person}(x) \land \text{Redhair}(x) \land (\forall y \text{Person}(y) \Rightarrow \text{Loves}(y, x)) \]

Equality

\( \text{term}_1 = \text{term}_2 \) is true under a given interpretation if and only if \( \text{term}_1 \) and \( \text{term}_2 \) refer to the same object

Examples

- Father(John)=Henry
- \( 1 = 2 \) is satisfiable
- \( 2 = 2 \) is valid
- Useful to distinguish two objects:
  - Definition of (full) \textit{Sibling} in terms of \textit{Parent}:
  \[ \forall x, y \text{Sibling}(x, y) \Leftrightarrow [\neg(x = y) \land \exists m, f \neg(m = f) \land \text{Parent}(m, x) \land \text{Parent}(f, x) \land \text{Parent}(m, y) \land \text{Parent}(f, y)] \]
  - Spot has at least two sisters: ...

AIMA, Exercise 8.4 & 8.7
Knowledge representation (KR)

**Domain:** a section of the world about which we wish to express some knowledge

**Example:** Family relations (kinship):
- Objects: people
- Properties: gender, married, divorced, single, widowed
- Relations: parenthood, brotherhood, marriage...

**Unary predicates:** Male, Female

**Binary relations:** Parent, Sibling, Brother, Child, etc.

**Functions:** Mother, Father

∀ m, c, Mother(c) = m ⇔ Female(m) ∧ Parent(m, c)

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**In Logic** (informally)

- Basic facts: **axioms**
- Derived facts: **theorems**

**Independent axiom**

an axiom that cannot be derived from the rest

→ Goal of mathematicians: find the minimal set of independent axioms

**In AI**

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK
Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([\text{Smell, Breeze, None}], 5))$
$Ask(KB, \exists a\text{Action}(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: Yes, $\{a/\text{Shoot}\}$ ← substitution (binding list)

Given a sentence $S$ and a substitution $\sigma$,
$S\sigma$ denotes the result of plugging $\sigma$ into $S$; e.g.,

$S = \text{Smarter}(x, y)$
$\sigma = \{x/\text{Hillary}, y/\text{Bill}\}$
$S\sigma = \text{Smarter}(\text{Hillary, Bill})$

$Ask(KB, S)$ returns some/all $\sigma$ such that $KB \models S\sigma$

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Prepare for next lecture: AIMA, Exercise 8.6, page 268

$Takes(x, c, s)$: student $x$ takes course $c$ in semester $s$

$Passes(x, c, s)$: student $x$ passes course $c$ in semester $s$

$Score(x, c, s)$: the score obtained by student $x$ in course $c$ in semester $s$

$x > y$: $x$ is greater that $y$

$F$ and $G$: specific French and Greek courses

$Buys(x, y, z)$: $x$ buys $y$ from $z$

$Sells(x, y, z)$: $x$ sells $y$ from $z$

$Shaves(x, y)$: person $x$ shaves person $y$

$Born(x, c)$: person $x$ is born in country $c$

$Parent(x, y)$: person $x$ is parent of person $y$

$Citizen(x, c, r)$: person $x$ is citizen of country $c$ for reason $r$

$Resident(x, c)$: person $x$ is resident of country $c$ of person $y$

$Fools(x, y, t)$: person $x$ fools person $y$ at time $t$

$Student(x)$, $Person(x)$, $Man(x)$, $Barber(x)$, $Expensive(x)$, $Agent(x)$, $Insured(x)$, $Smart(x)$, $Politician(x)$,
AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986
(then: University of Rochester
then: head of AI at AT&T Labs-Research
and Program co-chair of AAAI-2000
Now: Associate Professor at University of Washington, Seattle)