CSCE476/876 **Spring 2008**

Homework 6

Assigned on: Monday, March 24, 2008.

Due: Monday, March 31, 2008.

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This is a pen-and-paper homework, to be returned in class
The whole homework is worth 56 points

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1 Scheduling problem as a CSP (10 points)

Courtesy of Rina Dechter, UC-Irvine.

Consider the problem of scheduling five tasks: T1, T2, T3, T4 and T5, each of which takes one hour to complete. The tasks may start at 1:00, 2:00 or 3:00. Any number of tasks can be executed simultaneously provided the following restrictions are satisfied. T1 must start after T3. T3 must start before T4 and after T5. T2 cannot execute at the same time as T1 or T4. And T4 cannot start at 2:00.

1. Formulate the problem as a CSP by stating: the variables, their values and the constraints (hint: focus on the start time of a task.)
2. Draw the constraint graph.
3. Apply arc-consistency to each constraint in the CSP until no values can be ruled out (the CSP becomes arc-consistent).

2 Consistency checking (Total 10 points)

Consider the 4-Queens problem where each queen is associated with a row and can be assigned to any column in the row.

1. Express the domains of the variables.
2. Consider the binary constraints between every two queens and make explicit the constraint definition (i.e., give the constraint definition in extension).
3. Draw the constraint graph.
4. Arc-consistency of a binary constraint C_{Q_i,Q_j} between variables Q_i and Q_j ensures that every value for the variable Q_i has a support (at least one consistent value) in the domain of the variable Q_j, and vice versa. Run manually arc-consistency on the 4-Queens problem. Can you remove any value? At the end of the operation, the CSP is said to be arc-consistent.
5. Arc-consistency is also called 2-consistency because it considers all combinations of two variables at the same time. Let's consider *all combinations* of 3 variables at the same time. A CSP is said to be 3-consistent, if for every combination of consistent values for any two variables, one can find a value in the domain of the third variable such that the constraints between the three variables are satisfied. Will applying 3-consistency to the 4-queens problem remove any values?
6. What can you conclude about the effectiveness of consistency algorithms for the 4-Queens problem?

3 Consistency checking (10 points)

Courtesy of Rina Dechter, UC-Irvine.

Consider the simple coloring problem shown in Figure 1.

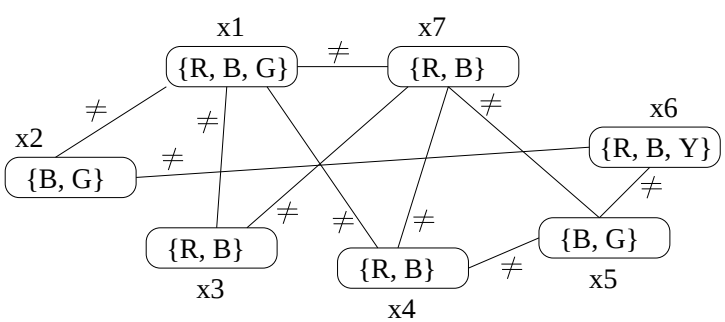

Figure 1: *A simple CSP.*

1. Compute the equivalent network that is *strong* 3-consistent
2. Find a solution to the CSP.

4 Latin square (10 points)

Adapted from of Daphne Koller, Stanford University.

A Latin Square is a $N \times N$ array filled with colors, in which no color appears more than once in any row or column. Finding a solution to a 4×4 Latin Square can be formulated as a CSP, with a variable for each cell in the array, each having a domain of $\{r, g, b, y\}$, and a set of constraints asserting that any pair of cells appearing in the same row must have different colors, and that any pair of cells appearing in the same column must have different colors.

1 **r**	2 **g**	3 **b**	4 **y**
5 **g**	6 **y**	*r g* *b y*	r g b y
7 **b**	r g b y	r g b y	r g b y
r g b y	r g b y	r g b y	r g b y

Figure 2: *Current state of search.*

At this point in the search, seven of the cells have been instantiated (displayed in **boldface** in Figure 2), and the initial domains of the remaining cells are shown. (The next cell to instantiate has the domain values in *italics*, but this information is irrelevant for this exercise.)

Re-execute (from scratch) the same 7 first assignments in the specified order and, at <u>*each*</u> assignment, draw the Latin Square while filtering the domains of the relative future variables. Do this process for the two following look-ahead strategies:

- the partial look-ahead strategy, forward checking (FC)
- the full look-ahead strategy, maintaining arc consistency.

Indicate eliminated values by crossing them out or just erasing them.

5 (Informal) reduction of 3SAT into a CSP (8 points)

Explain how to map any 3SAT instance into a CSP.

Indications:

- Your formulation should be as general as possible and should represent each of the elements of the 3SAT and its question in the terminology of a CSP.
- Consider X, the set of Boolean variables of a 3SAT instance. What are the values that a variable can take? Use this to define the variables of hte CSP and their values.

- A clause is a disjunction of literals. How to represent a clause in the CSP forumalism?
- A 3SAT sentence is a conjunction of clauses. How is the sentence represented in the CSP formalism?
- Finally, state how the question of 3SAT is reduced as a question to the CSP.

6 Dynamic Variable Ordering (Total 8 points)

1. Explain the principle for dynamic variable ordering in Backtrack Search. (2 points)
2. Describe three heuristics that impliment this principal. (6 points)