Title: On the Conversion between Non-Binary and Binary Constraint Satisfaction Problems
Authors: F. Bacchus and P. van Beek
Proc: AAAI 1998
Pages: 310–319

Foundations of Constraint Processing
CSCE421/821, Spring 2008
`www.cse.unl.edu/~choueiry/S08-421-821/`

Berthe Y. Choueiry (Shu-we-ri)
Avery Hall, Room 123B
`choueiry@cse.unl.edu`, Tel: (402)472-5444

Required reading:
On the Conversion between Non-Binary and Binary Constraint Satisfaction Problems, F. Bacchus and P. van Beek (AAAI'98)

Recommended reading: n-FC *available from course URL*

- On forward checking for non-binary constraint satisfaction.
  C. Bessière and P. Meseguer and E.C. Freuder and J. Larrosa,
  Proceedings CP'99, Alexandria VA, pages 88-102.
- Decomposable Constraints.
  Ian Gent, Kostas Stergiou and Toby Walsh.
  Artificial Intelligence, 123 (1-2), 133-156, 2000.

Summary

- Studies 2 mappings of non-binary CSPs into a binary representation $\begin{cases} \text{dual graph} \\ \text{hidden variable} \end{cases}$
- Studies performance of BT search in each mapping vs. its performance in non-binary version
- Considers theoretical & experimental aspects
- Proposes FC^+, yet lookahead strategy

Our goal: Learn about the mappings

Facts

- Non-binary constraints useful in the modeling of many applications
- Most research in CSPs is restricted to binary constraints
- Generalizing techniques for binary CSPs to address non-binary constraints is not straightforward

  .. but sometimes done: FC & MAC
- Projection looses information
- Usual work-around/justification: (correctly) map non-binary constraints into binary ones

Ideally

- Modeling: use the most expressive/natural representation
- Solving: use the most 'effective' representation

PS: the 'effectiveness' of a **representation** per se is a new, and difficult, research area. No clear metrics exist, to my knowledge

Your options

- Directly apply techniques for non-binary CSP
  ...too few :—(
- Translate non-binary→binary, then solve
  Techniques for binary CSPs exploit graph/constraint properties
  Does the translation preserve/yield such properties?
  ...will the translation degrade the performance of the techniques developed for binary CSPs?

Goal

- Study the effect of the translation on the performance of BT search
- Ultimately, establish properties of the translation to legitimize the restriction of research efforts to binary CSPs

Considers two translation methods

Results

- In most cases, the non-binary representation is most effective
- For tight constraints: binary representation wins

Example:

3SAT:
$(X_1 \vee X_2 \vee X_6) \wedge (\bar{X_1} \vee X_3 \vee X_4) \wedge (\bar{X_4} \vee \bar{X_5} \vee X_6) \wedge (X_2 \vee X_5 \vee \bar{X_6})$

3SAT as a non-binary (ternary) CSP

Variables: X_1, X_2, ..., X_6
Domains: $D_{X_i} = \{0, 1\}$
Constraints: $C_{126} = \{(0,0,1), (0,1,0), \ldots\}$, except (0,0,0)
$C_{134} = \{0,1\} \times \{0,1\} \times \{0,1\} \setminus \{(1,0,0)\}$
$C_{456} = \{0,1\} \times \{0,1\} \times \{0,1\} \setminus \{(1,1,0)\}$
$C_{256} = \{0,1\} \times \{0,1\} \times \{0,1\} \setminus \{(0,0,1)\}$

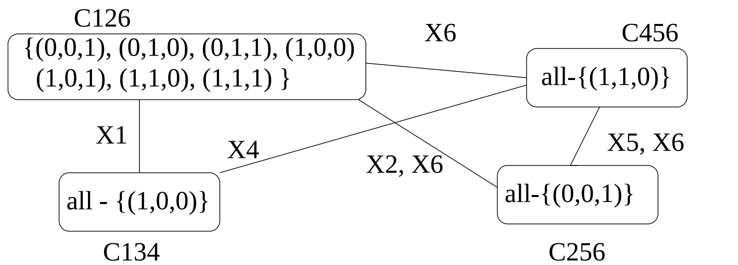
FC for non-binary constraints

- A k-ary constraint is forward-checkable, if
  - ($k-1$) of its variables are instantiated
  - one variable uninstantiated
- BT-search:
  - instantiate one variable
  - repeat: for each newly f-checkable constraint, check future variable
  - if any domain is empty, backtrack
- Improvements: n-FC, n-FC2, ..., n-FC5

Dual-graph representation

Usually: $\begin{cases} \text{CSP variable} \rightarrow \text{node} \\ \text{constraint} \rightarrow \underline{\text{hyper}}\text{-arc 'label'} \end{cases}$

Dual graph: $\begin{cases} \text{constraint} \rightarrow \text{node (called c-variable)} \\ \text{CSP variable} \rightarrow \underline{\text{arc}} \text{ 'label'} \end{cases}$



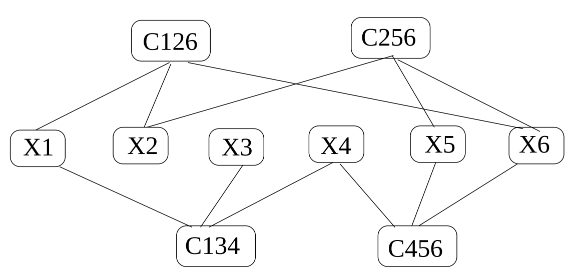
Constraint: X_1 must have the same value in C_{126} and C_{134}

Domain of a c-variable: constraint definition

Hidden-variable representation

Variables: CSP variables +
1 hidden variable (h-variable) per constraint

Constraints: only between a variable and the h-variables
corresponding to its applicable constraints

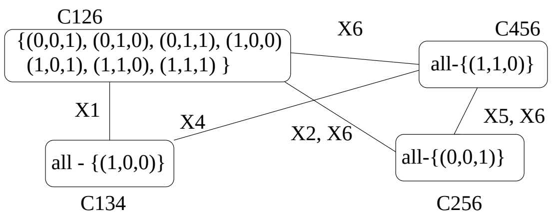


Constraint: a value of C_{126} correspond to one value of X_1

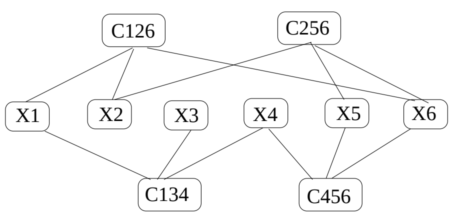
Domain of the h-variable = domain of the c-variable

Two binary representations

- **Dual graph**
  Nodes = only the constraints
  (CSP variables are not represented)
  Simple arcs between constraints



- **Hidden variable**
  Nodes = CSP variables and constraints
  Simple arcs constraints ⟷ variables



⟶ Compare to Freuder's constraint graphs

Theoretical comparison

I– Space requirements (data structures)

II– Analytical bounds (#nodes, #constraint checks in search)

I– Space requirements

- Binary representations require additional storing of domains for the c/h-variables (allowed k-tuples for each k-ary constraint)

  FC needs storage space proportional to the size of the domains (i.e., reductions)

  → could be substantial

- No space is needed to store constraints in binary representations: simple projection of an instantiation, can be done in constant time **assuming** domains of c/h-variables are stored extensionally

II– Analytical Bounds

Criteria

- number of visited nodes
- number of checks performed

Working assumption

- checking k-constraint costs k operations
- checking binary constraint costs 2 operations

Comparison

- dual-graph vs. non-binary
- hidden-variable vs. non-binary

Result

- not conclusive (one can always build a case where solving BT+FC has a better performance in one representation than in another)
- experimental evidence needed

Dual graph vs. non-binary CSP (I)

Loose constraint ⇒ exponentially large domains for c-variables ⇒ non-binary is less costly

Example:
n variables: $X_1, X_2, \ldots X_n$
n constraints: $X_1, \bar{X_1} \vee X_2, \bar{X_1} \vee \bar{X_2} \vee X_3, \ldots, \bar{X_1} \vee \bar{X_1} \vee \ldots X_n$
Non-binary: n nodes, $\mathcal{O}(n^2)$ consistency checks
Dual-graph: n nodes, $\mathcal{O}(2^n)$ consistency checks

Tight constraint ⇒ … ⇒ dual-graph is less costly

Example:
n variables: $X_1, X_2, \ldots X_n$
n constraints: $X_1 \wedge \ldots \wedge X_{n-1}, X_1 \wedge \ldots \wedge X_{n-2} \wedge X_n, \ldots, X_2 \wedge \ldots \wedge X_n$
Non-binary: 2^{n-1} nodes, $\mathcal{O}(n2^n)$ consistency checks
Dual-graph: n nodes, $\mathcal{O}(n^2)$ consistency checks

Improving FC: FC$^+$

- The constraint in the direction hidden-var→CSP-var is functional, but not vice-versa
- Search on hidden-var representation is restricted to the CSP-vars, h-vars used only for propagation
- FC is replaced with FC$^+$ to improve propagation
- FC$^+$ triggered improvements into nFC0, nFC1, …, nFC5.

Experiments

Carried out on random CSPs

Results have predictive power verified by:
- random 3SAT
- crossword puzzles

Conclusions

Translating non-binary constraints involves overhead.
Translation is **perhaps** worthwhile if constraints are restrictive
Translation, as a strategy, is justifiable

Many open issues..

→ # tuples in constraints a good indicator? probably..
→ dual graph vs. hidden-variable ?
→ .. we need to study further these translations/reformulations
→ to gain insight for designing good algorithms for non-binary constraints