Title: First-Order Logic

AIMA: Chapter 8 (Sections 8.1 and 8.2)

Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence

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## Outline

- First-order logic:
  - basic elements
  - syntax
  - semantics
- Examples

**Pros and cons** of propositional logic

- Propositional logic is declarative: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
  (unlike most data structures and databases)
- Propositional logic is compositional:
  meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent
  (unlike natural language, where meaning depends on context)
- but...
  Propositional logic has very limited expressive power
  E.g., cannot say "pits cause breezes in adjacent squares"
  except by writing one sentence for each square







## Propositional Logic

- is simple
- illustrates important points:
  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness:
  → In PL, world contains facts

## First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)

## First Order Logic

→ FOL provides more "primitives" to express knowledge:
- — objects (identity & properties)
- — relations among objects (including functions)

**Objects:** people, houses, numbers, Einstein, Huskers, event, ..
**Properties**: smart, nice, large, intelligent, loved, occurred, ..
**Relations**: brother-of, bigger-than, part-of, occurred-after, ..
**Functions**: father-of, best-friend, double-of, ..

**Examples**: (objects? function? relation? property?)
— one plus two equals four [sic]
— squares neighboring the wumpus are smelly







## Logic

**Attracts**: mathematicians, philosophers and AI people

**Advantages:**
— allows to represent the world and reason about it
— expresses anything that can be programmed

**Non-committal to**:
— symbols could be objects or relations
(*e.g.*, King(Gustave), King(Sweden, Gustave), Merciless(King))
— classes, categories, time, events, uncertainty

**.. but amenable** to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

⟶ Some people think FOL *is* the language of AI
true/false? donno :—( but it will remain around for some time..

# Types of logic

Logics are characterized by what they commit to as "primitives"

**Ontological commitment** :

what exists—facts? objects? time? beliefs?

**Epistemological commitment** :

what states of knowledge?

| Language | Ontological Commitment (What exists in the world) | Epistemological Commitment (What an agent believes about facts) |
|---|---|---|
| Propositional logic | facts | true/false/unknown |
| First-order logic | facts, objects, relations | true/false/unknown |
| Temporal logic | facts, objects, relations, times | true/false/unknown |
| Probability theory | facts | degree of belief 0…1 |
| Fuzzy logic | degree of truth | degree of belief 0…1 |

Higher-Order Logic: views relations and functions of FOL as objects







## **Syntax of FOL**: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: $x$, $y$, etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation) Father-of, Square-root, LeftLeg, etc.
- Quantifiyers $\forall$, $\exists$
- Connectives: $\wedge$, $\vee$, $\neg$, $\Rightarrow$, $\Leftrightarrow$,
- (Sometimes) equality $=$

Predicates and functions can have any arity (number of arguments)

**Basic elements** in FOL (i.e., the grammar)

**In propositional logic**, every expression is a sentence

**In FOL**,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier







**Term**

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

$$\begin{aligned} \text{Term} &= \mathit{function}(\mathit{term}_1, \ldots, \mathit{term}_n) \\ &\quad \text{or constant or variable} \end{aligned}$$

— **ground term**: term with no variable

## Atomic sentences

state facts

built with terms and predicate symbols

$$\text{Atomic sentence} = \mathit{predicate}(\mathit{term}_1, \ldots, \mathit{term}_n) \text{ or } \mathit{term}_1 = \mathit{term}_2$$

**Examples**:

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))







## Complex Sentences

built with atomic sentences and logical connectives

$\neg S$

$S_1 \wedge S_2$

$S_1 \vee S_2$

$S_1 \Rightarrow S_2$

$S_1 \Leftrightarrow S_2$

**Examples**:

Sibling(KingJohn,Richard) $\Rightarrow$ Sibling(Richard,KingJohn)

$>(1,2) \vee \leq(1,2)$

$>(1,2) \wedge \neg{>}(1,2)$

13



## **Truth in first-order logic**: Semantic

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for
*constant symbols* → objects
*predicate symbols* → relations
*function symbols* → functional relations

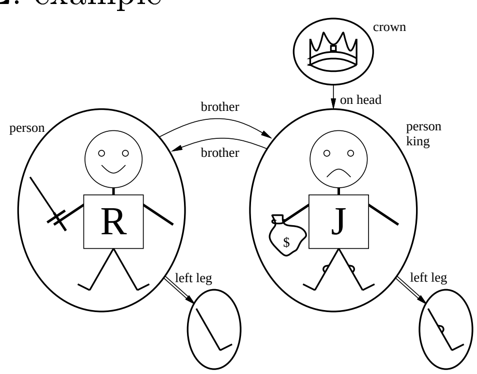
An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by *predicate*



14



## **Model in FOL**: example




The domain of a model is the set of objects it contains:
five objects

Intended interpretation: Richard refers Richard the Lion Heart, John refers to Evil King John, Brother refers to brotherhood relation, etc.

## **Models for FOL:** Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements $n$ from 1 to $\infty$
  For each $k$-ary predicate $P_k$ in the vocabulary
    For each possible $k$-ary relation on $n$ objects
      For each constant symbol $C$ in the vocabulary
        For each choice of referent for $C$ from $n$ objects ...

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded
$\longrightarrow$ Checking entailment by enumerating is not an option







## Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about <u>everything</u>
- existential quantifier: make statements about <u>some things</u>

## Universal quantification

$\forall \langle variables \rangle \ \langle sentence \rangle$

**Example**: all dogs like bones $\forall\, x Dog(x) \Rightarrow LikeBones(x)$

x = Indy is a dog x = Indiana Jones is a person

$\forall\, x\, P$ is equivalent to the <u>conjunction</u> of <u>instantiations</u> of $P$

$$\begin{array}{ll} & Dog(Indy) \Rightarrow LikeBones(Indy) \\ \wedge & Dog(Rebel) \Rightarrow LikeBones(Rebel) \\ \wedge & Dog(KingJohn) \Rightarrow LikeBones(KingJohn) \\ \wedge & \ldots \end{array}$$

**Typically**: $\Rightarrow$ is the main connective with $\forall$

**Common mistake**: using $\wedge$ as the main connective with $\forall$

Example: $\forall\, x \ \ Dog(x) \ \wedge \ LikeBones(x)$

all objects in the world are dogs, and all like bones







## Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

**Example**: some student will talk at the TechFair

$\exists\, x Student(x) \wedge TalksAtTechFair(x)$

Pat, Leslie, Chris are students

$\exists\, x\, P$ is equivalent to the <u>disjunction</u> of <u>instantiations</u> of $P$

$$\begin{array}{ll} & Student(Pat) \ \wedge TalksAtTechFair(Pat) \\ \vee & Student(Leslie) \wedge TalksAtTechFair(Leslie) \\ \vee & Student(Chris) \wedge TalksAtTechFair(Chris) \\ \vee & \ldots \end{array}$$

**Typically**: $\wedge$ is the main connective with $\exists$

**Common mistake**: using $\Rightarrow$ as the main connective with $\exists$

$\exists\, x \ Student(x) \ \Rightarrow \ TalksAtTechFair(x)$

is true if there is anyone who is not Student

## Properties of quantifiers (I)

$\forall x \, \forall y$ is the same as $\forall y \, \forall x$

$\exists x \, \exists y$ is the same as $\exists y \, \exists x$

$\exists x \, \forall y$ is not the same as $\forall y \, \exists x$

$\exists x \, \forall y \, Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y \, \exists x Loves(x, y)$
"Everyone in the world is loved by at least one person"

**Quantifier duality**: each can be expressed using the other

$\forall x \, Likes(x, IceCream) \qquad \neg \, \exists x \, \neg Likes(x, IceCream)$

$\exists x \, Likes(x, Broccoli) \qquad \neg \, \forall x \, \neg Likes(x, Broccoli)$

**Parsimony principal**: $\forall$ , $\neg$, and $\Rightarrow$ are sufficient







## Properties of quantifiers (II)

**Nested quantifier**:
$\forall \, x (\exists \, y (P(x, y))$:
every object in the world has a particular property, which is the property to be related to some object by the relation P

$\exists \;\; x \; (\forall \, y(P(x, y))$:
there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

**Lexical scoping:** $\forall \, x[Cat(x) \vee \exists \, x Brother(Richard, x)]$

**Well-formed formulas (WFF)**: (kind of correct spelling)
every variable must be introduced by a quantifier
$\forall \, x P(y)$ is not a WFF

## Examples

Brothers are siblings

.

"Sibling" is symmetric

.

One's mother is one's female parent

.

A first cousin is a child of a parent's sibling







## Examples

.

$\forall x, y \; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$

.

$\forall x, y \; Sibling(x, y) \;\Rightarrow\; Sibling(y, x)$

.

$\forall x, y \; Mother(x, y) \;\Rightarrow\; (Female(x) \;\wedge\; Parent(x, y))$

.

$\forall x, y \; FirstCousin(x, y) \Leftrightarrow$
$\exists a, b \; Parent(a, x) \wedge Sibling(a, b) \wedge Parent(b, y)$

## Tricky example

Someone is loved by everyone

$\exists x \, \forall y \; Loves(y, x)$

Someone with red-hair is loved by everyone

$\exists x \, \forall y \; Redhair(x) \land Loves(y, x)$

Alternatively:

$\exists x \; Person(x) \land Redhair(x) \land (\forall y \; Person(y) \Rightarrow Loves(y, x))$







## Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

Examples

- Father(John)=Henry
- $1 = 2$ is satisfiable
- $2 = 2$ is valid
- Useful to distinguish two objects:
  – Definition of (full) *Sibling* in terms of *Parent*:
  $\forall x, y \; Sibling(x, y) \Leftrightarrow [\neg(x = y) \land \exists m, f \neg(m = f) \land Parent(m, x) \land Parent(f, x) \land Parent(m, y) \land Parent(f, y)]$
  – Spot has at least two sisters: ...

AIMA, Exercise 8.4 & 8.7

## Knowledge representation (KR)

**Domain**: a section of the world about which we wish to express some knowledge

**Example**: Family relations (kinship):
– Objects: people
– Properties: gender, married, divorced, single, widowed
– Relations: parenthood, brotherhood, marriage..

**Unary predicates**: Male, Female
**Binary relations**: Parent, Sibling, Brother, Child, etc.
**Functions**: Mother, Father

$\forall\, m, c, Mother(c) = m \;\Leftrightarrow\; Female(m) \land Parent(m, c)$







## In Logic (informally)

- Basic facts: <u>**axioms**</u> (definitions)
- Derived facts: <u>**theorems**</u>

**Independent axiom**
an axiom that cannot be derived from the rest
$\longrightarrow$ Goal of mathematicians: find the minimal set of independent axioms

## In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK

## Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$
$Ask(KB, \exists a Action(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: $Yes$, $\{a/Shoot\}$ ← <u>substitution</u> (binding list)

Given a sentence $S$ and a substitution $\sigma$,
$S\sigma$ denotes the result of plugging $\sigma$ into $S$; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$Ask(KB, S)$ returns some/all $\sigma$ such that $KB \models S\sigma$







**Prepare for next lecture:** AIMA, Exercise 8.6, page 268

Takes$(x, c, s)$: student $x$ takes course $c$ in semester $s$

Passes$(x, c, s)$: student $x$ passes course $c$ in semester $s$

Score$(x, c, s)$: the score obtained by student $x$ in course $c$ in semester $s$

$xy$: $x$ is greater that $y$

$F$ and $G$: specific French and Greek courses

Buys$(x, y, z)$: $x$ buys $y$ from $z$

Sells$(x, y, z)$: $x$ sells $y$ from $z$

Shaves$(x, y)$: person $x$ shaves person $y$

Born$(x, c)$: person $x$ is born in country $c$

Parent$(x, y)$: person $x$ is parent of person $y$

Citizen$(x, c, r)$: person $x$ is citizen of country $c$ for reason $r$

Resident$(x, c)$: person $x$ is resident of country $c$ of person $y$

Fools$(x, y, t)$: person $x$ fools person $y$ at time $t$

Student $(x)$, Person$(x)$, Man$(x)$, Barber$(x)$, Expensive$(x)$, Agent$(x)$,
Insured$(x)$, Smart$(x)$, Politician$(x)$,

## AI Limerick

If your thesis is utterly vacuous

Use first-order predicate calculus

With sufficient formality

The sheerest banality

Will be hailed by the critics: "Miraculous!"

*Henry Kautz*

*In Canadian Artificial Intelligence, September 1986*

*(then: University of Rochester*

*then: head of AI at AT&T Labs-Research*

*and Program co-chair of AAAI-2000*

*Now: Associate Professor at University of Washington, Seattle)*