8.6 Linear inequalities

CSCE990-06
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Outline

- We'll examine convexity and bucket algorithms for linear inequalities.

Linear elimination

<u>**Linear Inequality**</u> $\sum_{i=1}^{r} a_i x_i \leq c_i$

<u>**Example**</u>

What do we notice?

Linear Elimination

Linear Elimination Let $\alpha = \sum_{i=1}^{r} a_i x_i \leq c_i$. Let $\beta = \sum_{i=1}^{r} b_i x_i \leq d_i$. Then $elim_r(\alpha, \beta)$ is applicable only if a_r and b_r have opposite signs, in which case:
$elim_r(\alpha, \beta) = \sum_{i=1}^{r-1} (-a_i \frac{b_r}{a_r} + b_i) x_i \leq -\frac{b_r}{a_r} c_i + d_i$.
If a_r and b_r have the same sign, the elimination implicitly generates the universal constraint.

Why?

How does this relate?

With finite domains, linear inequalities can be written as extensional relations. With rational domains, relations can be kept as inequalities.

Then, we can enforce relational path consistency via EC_2 just as we did with other convex relations. Remember, this can solve the system — it's convex. With rational domains, use the elimination form of EC_2.

Lemma 8.6.2 EC_2 and linear elimination.
$sol(elim_r(\alpha, \beta)) \supseteq EC_r(sol(\alpha), sol(\beta))$
when the domains are the integers.
However, over the rationals,
$sol(elim_r(\alpha, \beta)) = EC_r(sol(\alpha), sol(\beta))$

Fourier Bucket Elimination

$\underline{DRC_2}$ **modified:** Using the same directional bucket algorithm as for row-convex relations, we can generate backtrack-free representations with rational (and even real) domains:

1. Choose an ordering. Fill buckets with the inequalities. Then in reverse order . . .

2. If $domain_i$ has one value left, substitute it into the bucket's inequalities and put the results into the next bucket(s).

3. Otherwise, perform EC_2 (using elimination for intensional relations and normal EC_2 for extensional).

4. Return the union of all the buckets (domains), or nil if elimination annihilates a domain.

Fourier Elimination Example

Let $\varphi = \begin{cases} -x_1 & +3x_2 & & +5x_4 & \leq & 5 \\ x_1 & & & +x_4 & \leq & 2 \\ & & & -x_4 & \leq & 0 \\ & & x_3 & & \leq & 5 \\ x_1 & +x_2 & -x_3 & & \leq & -10 \\ x_1 & +2x_2 & & & \leq & 0 \end{cases}$

Summary

Problems with special domains can be solved by enforcing relational path (2-) consistency (RPC). The generic algorithms used are RC_2 or DRC_2, both of which achieve RPC via extended 2-composition, EC_2.

Each domain type has a version of EC_2 suitable for that domain.

plain EC_2 for discrete, extensional relations, esp. those with row-convex domains.

resolution for boolean domains.

elimination for linear inequalities.