8.5 Row Convex Constraints

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Eric Moss
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Outline

- In the case of *row-convex* constraints, relational path consistency guarantees global consistency.

- Thus we should search for row-convexity where we can.

Definitions

Functional A binary relation R_{ij} expressed as a (0,1) matrix is *functional* if and only if there is at most a single "1" in any row or column of R_{ij}.

Monotone Given some ordering of the domain of values for all variables, a binary relation R_{ij} expressed as a (0,1) matrix is *monotone* if the following conditions hold: if $(a, b) \in R_{ij}$ and $c \geq a$, then $(c, b) \in R_{ij}$, and if $(a, b) \in R_{ij}$ and $c \leq b$, then $(a, c) \in R_{ij}$.

Row Convex A binary relation R_{ij} represented as a (0,1) matrix is *row convex* if in each row (column) all of the ones are consecutive; that is, no two ones within a single row are separated by a zero in that same row (column).

Examples

Functional

Monotone

Row Convex

(0,1) Representations

Arc-consistency looks like:

Path-consistency looks like:

Convexity & Intersection

<u>Lemma 8.5.4</u> Let F be a finite collection of (0,1)-row vectors that are row convex and of equal length such that every pair of row vectors in F have a non-zero entry in common; that is, their intersection is not the vector with all zeros. Then all of the row vectors in F have a non-zero entry in common.

Convexity & Consistency

Theorem 8.5.5 Let R be a path consistent binary constraint network. If there exists an ordering of the domains of R such that the relations of all constraints are row convex, the network is globally consistent and is therefore minimal.

- For every set of k variables, arc-consistency forces every row in every $R_{i,k}$ to have a non-zero entry.

- Path consistency forces all those rows to share some non-zero entry with some other row.

- Convexity forces all those rows to share the *same* entry or entries.

Theorem 8.5.5 (cont'd)

- All $R_{i,k}$ sharing a non-zero entry means there is some value from D_k consistent with all the relations among all the relations between the k domains. Thus the network is k-consistent. Since $1 \leq k \leq n$, the network is consistent.

Why is this useful?

- However we end up with path-consistency and row-convex constraints, the result is a consistent network.

- Thus, if enforcing PC creates row-convex constraints (like in Example 8.5.6), we end up with a consistent network.

- If we start with row-convex constraints, and they remain so under extended (2) composition, the result is consistent.

- Examples include linear programming problems in standard form (*not* with inequalities using $\neq$).

Note: Remember to condense (0,1) representations when domains get pruned.

Identifying Row-Convex Relations

Theorem 8.5.7 An $m \times n$ (0,1)-matrix with f non-zero entries can be tested for whether a permutation of the columns exists such that the matrix is row-convex in $O(m+n+f)$ steps.

Non-binary Row-Convex Relations

Definition 8.5.9 An r-ary relation R_S, where $S = \{x_1, \ldots, x_r\}$, is row convex if for any subset Z of $r-2$ variables $Z \subseteq S$ and for every instantiation, a, of Z, the binary relation $\pi_{(S-Z)}(\sigma_a(R))$ is row convex.

Identifying Row-Convex Relations (cont'd)

Theorem 8.5.10 Given a relationally path consistent network, if there exists an ordering of the domains such that the relations are row convex, the network is globally consistent.

Theorem 8.5.11 For any network whose closure under extended 2-composition is row convex, RC_2 will generate a globally consistent version of that network.

Proposition 8.5.12 A set of linear inequalities that is closed under RC_2 is globally consistent.

Identifying Row-Convex Relations (cont'd)

Theorem 8.5.13 For any network whose directional closure (given ordering d) relative to extended 2-composition is not empty and is also row convex, algorithm DRC_2 computes an equivalent network that is backtrack-free along the ordering.