# CSCE 976, Spring 2002 Lecture Notes for 03/11/2002 Second lecture on Golumbic's paper on Temporal Reasoning Scribe: Tibor Moldovan

**Paper:** "Reasoning About Time," Martin Ch. Golumbic. *Mathematical Aspects of Artificial Intelligence, Proc. Symp. On Applied Math.*, Vol 55, pages 19—53, 1998.

Golumbic notices from Allen's  $A_{13}$  that there are  $2^{13} - 1$  combinations of possible temporal relations.

Villain & Kautz already proved that satisfiability in  $A_{13}$  is NP-complete. They also show that satisfiability in  $P_3$  (point algebra with 3 relations) can be done in  $O(n^3)$ , and is thus tractable.

Golumbic then defines A<sub>3</sub> algebra (consisting of the relations: before, after and overlap) and proves that satisfiability for this restricted algebra remains NP-complete.

He identifies the following four important computational problems on interval algebra:

### I Satisfiability Problem (ISAT)

### II Minimal Labeling Problem (MLP)

Where every value in every 'vector' is present in at least one solution, in other words, if one removes one value, one solution is eliminated.

# **III All Consistent Solutions Problem (ACSP)**

Tries to represent all the solutions in a structure such that:

- 1. the size of the structure is polynomial,
- 2. the time to build the structure is polynomial.

# IV Endpoint Sequence Problems (ESP)

Which enumerates all distinct interval realizations that are consistent with the given data.

Kautz and Villain proved that MLP is NP-complete for  $A_{13}$  and that the path consistency algorithm is incomplete for MLP.

Golumbic shows that the above four problems are likely intractable (i.e., they are NP-complete or NP-hard) also for other algebras, such as  $A_3$ ,  $A_6$ ,  $A_7$ ,  $A_{13}$ 

Golumbic simplifies  $A_{13}$  algebra in two different ways:

- Keeps intervals as intervals, and takes subsets of relations, yielding A<sub>3</sub>, A<sub>6</sub>, A<sub>7</sub>
- Relaxes intervals, and uses time points, yielding P<sub>3</sub>

Furthermore, he simplifies  $A_3 \{<,>,\cap\}$  by taking away the combination <> (disjunction), thus making satisfiability linear.

Chapters 4 and 5 were briefly discussed as they deal with temporal logic and autonomous agents.

# **Student Questions and Comments:**

- Rob finds the paper somewhat confusing
- Praveen points out that in the construction site problem, a construction site could be a variable, not just an interval, thereby allowing for a more direct CSP approach.
- Shabbir points to page 10, example 7 and asks how did Golumbic come up with that intersection? The answer is that it is canonical, and it seems to be a conjunction of interval intersections.
- Shabbir also likes the application of the theory to practice, since most of the work being done on temporal reasoning has so far been theoretical.
- Xu Lin is disappointed that there is no future work section, and wonders about application of iterative repair strategies in temporal reasoning.
- Corey adds that he likes the simplification of the original  $A_{13}$  algebra into  $A_3$  and others.
- Dan thinks that Golumbic could be more descriptive with problems such as archeology, and bathroom problems (there is no representation given, and no solution).
- Amy is also disappointed that there is no answer to the boy-girl problem discussed in the text.