

**CSCE 976, Spring 2002**  
**Lecture Notes for 03/06/2002**  
**Tibor Moldovan**

**Paper:** “Reasoning About Time,” Martin Ch. Golumbic. *Mathematical Aspects of Artificial Intelligence, Proc. Symp. On Applied Math.*, Vol 55, pages 19—53, 1998.

**Regarding the previous lecture these three points were clarified:**

1. Contribution of the paper: Focus of the paper is to compare two representations (interval and point), describing how they relate, what each can express or cannot express.
2. Closure: If one derives everything that can be derived from a KB than one has closure.
3. Bit String Encoding: when a CSP is represented as a binary matrix the intersection of matrices can be treated as a logical AND, and the composition of matrices gives the matrix product.

**“Reasoning About Time”**

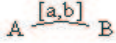
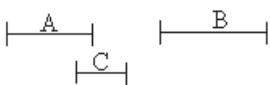
**Background:**

We can distinguish the following *dimensions* when discussing temporal reasoning systems

1. Time primitives (each with their own algebra):

Time points  
Time intervals

2. Types of relations: The relations among the primitives could be qualitative or quantitative. Qualitative relations specify non-numeric relations between points and/or intervals such as before, after, during, etc., while quantitative means that relations between the events specify numerical values such as distances.

	Qualitative	Quantitative
Time points	(< > =)	
Time intervals	Allen’s 13 relations	

If one has a reference point, one can have absolute points.

3. Emphasis on representation versus processing: Two approaches can be distinguished here:

*Logic* (e.g., event calculus) where focus is to develop a new logic-based language, (with a specific syntax and semantics) while using general inference mechanisms. Emphasis is made on problem representation.

*Algorithmic*: using graph theory and/or CSPs where the focus is to develop algorithms and this approach seems to be better when it actually comes to solving problems because it allows us to exploit the structure of the problem.

Some of the research issues addressed in the field are:

- Hybrids (using both intervals and time points, and quantitative and qualitative approaches at the same time)
- Methods for representing and handling persistence, and persistence clipping.
- **Time granularities** seem to be an area of difficulty, as there are inherited properties that move across hierarchies, so one has to keep track of different hierarchical levels while still being able to deal with time abstractions.

Golumbic identifies the following functionalities needed by a temporal reasoning system:

1. A KB with primitives for declaring time events and relating them
2. A procedure to enforce KB consistency
3. Routines for asking and answering queries

**Introducing structure:** Consistency enforcement can be done either globally, which requires lots of time, or with respect to intervals, or only when one makes an inquiry.

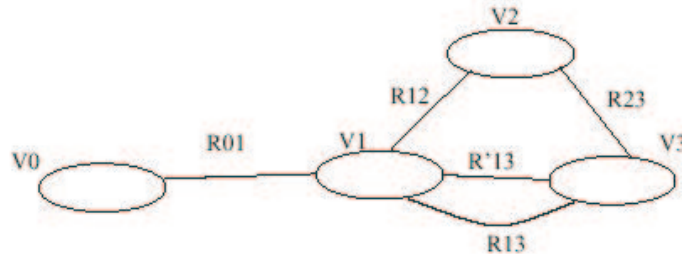
**Chronologies** (time windows) are also discussed in the paper. They deal with events that are not happening close to the intervals of interest, but their effects are present and affecting the present events.

### **Types of temporal problems and their complexity:**

1. *Metric*: Due to temporal interval disjunctions or ambiguities certain **complexities** arise which expand the expressive power of our language. Golumbic proposes a *metric* model that allows to express disjunctions: the metric temporal constraint problems (MTCP). MTCPs use regular interval timelines to record events and allow disjunctions of interval in the constraints. They are essentially temporal CSPs TCSPs of Dechter, Meiri and Pearl AIJ-1991. Another metric model, simple temporal problem (**STP**), was also discussed. Instead of concentrating on TCSP or MTCP one can focus on networks that don't allow disjunctions.
2. *Qualitative*: Author proposes **interval algebra** consisting of 3 relations: before  $<$ , after  $>$ , and overlap  $\cap$ .  
Golumbic and Shamir call it the  $A_3$  algebra. It is a subset of Allen's 13 relations.

We discussed the STP. Time points are transformed into a distance graph (a **DAG**, directed acyclic graph, with weights on the edges.) The advantage of this is that with

DAGs one can use powerful algorithms to solve them (Bellman-Ford, Floyd-Warshall, Johnson's etc.). If there are no negative cycles, the problem is consistent. The Floyd-Warshall algorithm is a special case of the Path Consistency algorithm obtained *whenever* the composition relation distributes over intersection [Montanari'74]. This property is verified in the temporal domain.



$$R_{01} \cdot (R_{13} \cap R'_{13}) = (R_{01} \cdot R_{13}) \cap (R_{01} \cdot R'_{13})$$

In Path Consistency, one has two operations: intersection  $\cap$ , and composition  $\bullet$ .  
When composition distributes over intersection then:

1. PC guarantees that CSP is minimal and decomposable
2. one loop is sufficient.

Transforming time points into a distance graph:

