Title: First-Order Logic

AIMA: Chapter 7 (Sections 7.1 and 7.3)

Introduction to Artificial Intelligence

CSCE 476-876, Spring 2001

URL: `www.cse.unl.edu/~choueiry/S01-476-876`

Berthe Y. Choueiry (Shu-we-ri)

`choueiry@cse.unl.edu`, `(402)472-5444`

Glossary 10:

Due date: Mon, April 9, 2000

Axioms
Function symbols
Ground term
Independent axiom
Predicate symbols
Quantifiers
Theorems
Tuples
WWF

Outline

- First-order logic:
  - basic elements
  - syntax
  - semantics
- Examples

Propositional Logic

- is simple
- illustrates important points:
  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)

First Order Logic

→ provides more "primitives" to express knowledge:
— objects (identity & properties)
— relations among objects (including functions)

Objects: people, houses, numbers, Einstein, Huskers, event, ..
Properties: smart, nice, large, intelligent, loved, occurred, ..
Relations: brother-of, bigger-than, part-of, occurred-after, ..
functions: father-of, best-friend, double-of, ..

Examples: (objects? function? relation? property?)
— one plus two equals four [sic]
— squares neighboring the wumpus are smelly

Logic

Attracts: mathematicians, philosophers and AI people

Advantages:

— allows to represent the world and reason about it

— expresses anything that can be programmed

Non-committal to:

— symbols could be objects or relations
(*e.g.*, King(Gustave), King(Sweden, Gustave))

— classes, categories, time, events, uncertainty

.. **but amenable** to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

⟶ Some people think FOL *is* the language of AI
true/false? donno :—(but it will remain around for some time..

Basic elements

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier

Term

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

$$\begin{aligned} \text{Term} \quad &= \quad function(term_1, \ldots, term_n) \\ &\quad\;\; \text{or constant or variable} \end{aligned}$$

— **ground term**: term with no variable

Atomic sentences

state facts

built with terms and predicate symbols

$$\begin{aligned} \text{Atomic sentence} \quad &= \quad \mathit{predicate}(\mathit{term}_1, \ldots, \mathit{term}_n) \\ & \quad\quad \text{or } \mathit{term}_1 = \mathit{term}_2 \end{aligned}$$

Examples:

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))

Complex Sentences

built with atomic sentences and logical connectives

$\neg S$

$S_1 \wedge S_2$

$S_1 \vee S_2$

$S_1 \Rightarrow S_2$

$S_1 \Leftrightarrow S_2$

Examples:

Sibling(KingJohn,Richard) $\Rightarrow$ Sibling(Richard,KingJohn)

$>(1, 2) \vee \leq(1, 2)$

$>(1, 2) \wedge \neg{>}(1, 2)$

Truth in first-order logic

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for
constant symbols → objects
predicate symbols → relations
function symbols → functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by *predicate*

Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about <u>everything</u>
- existential quantifier: make statements about <u>some things</u>

Universal quantification

$\forall \langle variables \rangle \;\; \langle sentence \rangle$

Example: all dogs like bones $\forall\, x Dog(x) \Rightarrow LikeBones(x)$

x = Indy is a dog x = Indiana Jones is a person

$\forall\, x\; P$ is equivalent to the <u>conjunction</u> of <u>instantiations</u> of P

$$\begin{array}{ll} & Dog(Indy) \Rightarrow LikeBones(KingJohn) \\ \wedge & Dog(Rebel) \Rightarrow LikeBones(Rebel) \\ \wedge & Dog(Rover) \Rightarrow LikeBones(Rover) \\ \wedge & \ldots \end{array}$$

Typically: $\Rightarrow$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$

Example: $\forall\, x \;\; Dog(x) \; \wedge \; LikeBones(x)$

all objects in the world are dogs, and all like bones

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Example: some top-student will attend AAAI

$\exists\, x TopStudent(x) \wedge AttendsAAAI(x)$

Pat, Leslie, Chris are top-students

$\exists\, x\ P$ is equivalent to the <u>disjunction</u> of <u>instantiations</u> of P

$$
\begin{array}{ll}
 & TopStudent(Pat) \wedge AttendsAAAI(Pat) \\
\vee & TopStudent(Leslie) \wedge AttendsAAAI(Leslie) \\
\vee & TopStudent(Chris) \wedge AttendsAAAI(Chris) \\
\vee & \ldots
\end{array}
$$

Typically: $\wedge$ is the main connective with $\exists$

Common mistake: using $\Rightarrow$ as the main connective with $\exists$

$\exists\, x\ TopStudent(x) \Rightarrow AttendsAAAI(x)$

is true if there is anyone who is not TopStudent

Properties of quantifiers (I)

$\forall x\ \forall y$ is the same as $\forall y\ \forall x$

$\exists x\ \exists y$ is the same as $\exists y\ \exists x$

$\exists x\ \forall y$ is <u>not</u> the same as $\forall y\ \exists x$

$\exists x\ \forall y\ Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y\ \exists x Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x\ Likes(x, IceCream) \qquad \neg\ \exists x\ \neg Likes(x, IceCream)$

$\exists x\ Likes(x, Broccoli) \qquad \neg\ \forall x\ \neg Likes(x, Broccoli)$

Parsimony principal: $\forall$, $\neg$, and $\Rightarrow$ are sufficient

Properties of quantifiers (II)

Nested quantifier:

$\forall\, x(\exists\, y(P(x,y))$:

every object in the world has a particular property, which is the property to be related to some object by the relation P

$\exists\;\; x\; (\forall\, y(P(x,y))$:

there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

Lexical scoping: $\forall\, x[Cat(x) \vee \exists\, xBrother(Richard, x)]$

Well-formed formulas (WFF): (kind of correct spelling)

every variable must be introduced by a quantifier

$\forall\, xP(y)$ is not a WFF

Examples

Brothers are siblings

.

"Sibling" is reflexive

.

One's mother is one's female parent

.

A first cousin is a child of a parent's sibling

Examples

.

$\forall x, y\ Brother(x, y) \ \Leftrightarrow \ Sibling(x, y)$

.

$\forall x, y\ Sibling(x, y) \ \Leftrightarrow \ Sibling(y, x)$

.

$\forall x, y\ Mother(x, y) \ \Leftrightarrow \ (Female(x) \ \wedge \ Parent(x, y))$

.

$\forall x, y\ FirstCousin(x, y) \Leftrightarrow$
$\exists a, b\ Parent(a, x) \wedge Sibling(a, b) \wedge Parent(b, y)$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

predicate, terms, and equality to form atomic sentences

Father(John)=Henry

AIMA, Exercise 7.4 & 7.5

Useful to distinguish two objects: Spot has at least two sisters

.

Knowledge representation (KR)

Domain: a section of the world about which we wish to express some knowledge

Example: Family relations (kinship):
– Objects: people
– Properties: gender, married, divorced, single, widowed
– Relations: parenthood, brotherhood, marriage..

Unary predicates: Male, Female
Binary relations: Parent, Sibling, Brother, Child, etc.
Functions: Mother, Father

$$\forall\, m, c, Mother(c) = m \;\Leftrightarrow\; Female(m) \wedge Parent(m, c)$$

In Logic (informally)

- Basic facts: **axioms** (definitions)
- Derived facts: **theorems**

Independent axiom
an axiom that cannot be derived from the rest
$\longrightarrow$ Goal of mathematicians: find the minimal set of independent axioms

In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK

Prepare for Monday:

AIMA, Exercise 7.2

Takes(x, H): student takes history course

Takes(x, B): student takes biology course

Fails(x, H): student fails history course

Fails(x, B): student fails biology course

Score(x, H): the score obtained by student x in history course

Score(x, B): the score obtained by student x in biology course

Disliked(x,y): person x dislikes person y

Shaves (x,y): person x shaves person y

Fools(x,y,t): person x fools person y at time t

Student (x), Person(x), Man (x), Woman(x), Barber (x),

Vegetarian(x), Smart(x), Professor(x), Politician(x),

AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986

(then: University of Rochester

now: head of AI at AT&T Labs-Research

Program co-chair of AAAI-2000)