Title: Solving Problems by Searching

AIMA: Chapter 3 (Sections 3.4 and 3.5)

Introduction to Artificial Intelligence

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List of terms for Glossary 3:

Given: Friday, Jan 26, 2001

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- Completeness
- Complexity (time/space)
- Diameter (of a graph). *Look in graph theory.*
- Expand (a search node)
- Fringe
- Optimality
- Search:
  - Breadth-first
  - Heuristic
  - Node
  - Space
  - Strategy
  - Tree
  - Uniformed (blind)

Summary

- Problem-solving agents

  Formulate, Search, Execute

- Formulating problems
  - Problem types:
    Simple-state, Multiple-State, Contingency, Exploration
  - Problem components: State, Operators, Goal test, Path cost
    Solution: sequence of actions from initial state to goal state
  - Importance of modeling

Today $\longrightarrow$ Search

Outline

- Generate action sequences
  - tree, node, fringe, queue.
  - Data structure
- Search strategies:
  - Four evaluation criteria.. ?
  - Six strategies for uniformed search

Search: generate action sequences

partial solution: sequence yielding a (non goal) intermediate state

Search $\left\{ \begin{array}{l} \text{generate} \\ \text{maintain} \end{array} \right\}$ a set of sequences of partial solutions

Two aspects:

1. how to generate sequences
2. which data structures to keep track of them

Search generate action sequences

Basic idea:

offline, simulated exploration of state space

by generating successors of already-explored states

$\rightarrow$ *expanding* states

Start from a state, test if it is a goal state

If it is, we are done

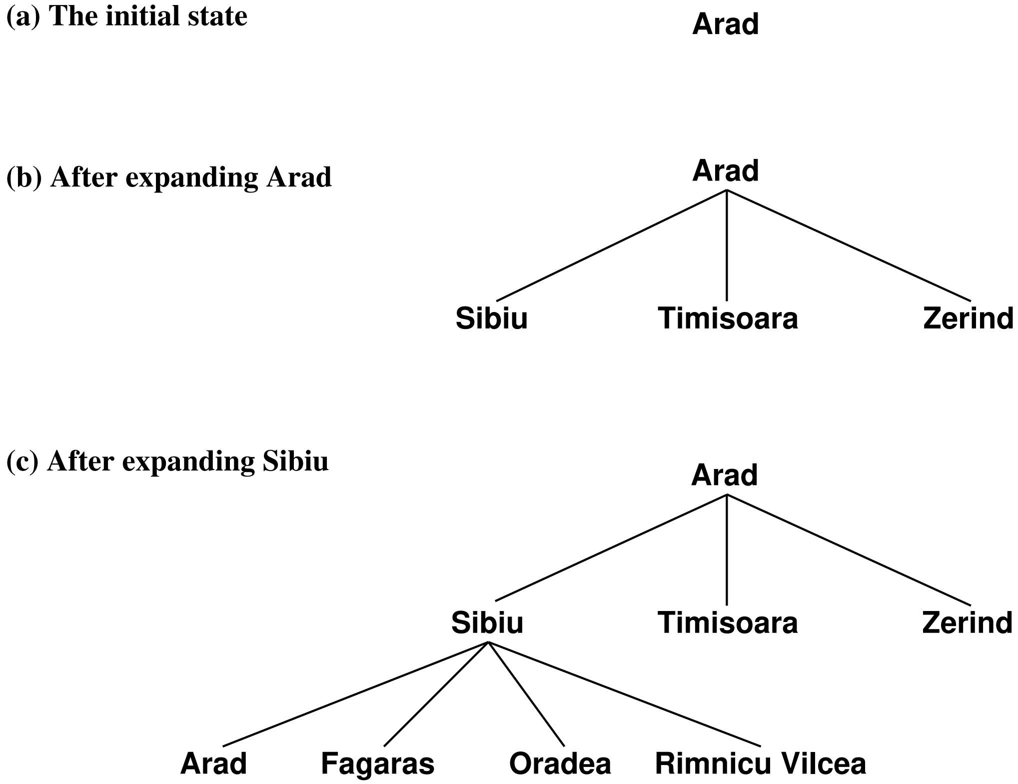
If it is not: *expand state*

Apply all operators applicable to current state to

generate all possible sequences of future states

now we have set of partial solutions

...



$$\text{Search tree, nodes} \begin{cases} \text{root: initial state} \\ \text{leaves: states that can/should not be expanded} \end{cases}$$

Data structure

LHW Chapter 13

for a search node:

- state in space state
- parent node
- operator used to generate node
- **depth**: # nodes from root (path length)
- path cost $g(x)$

A node x has a parent, children, depth, path cost $g(x)$

Nodes to be expanded:

- constitute a fringe (frontier)
- managed in a queue

Warning:

Do not confuse: <u>State</u> space and <u>Search</u> (tree) space

Holiday in Romania:

- What is a state?
- What is the state space?
- What is the size of state space?
- What is the size of search tree ?

A node has a parent, children, depth, path cost $g(x)$

A state has no parent, children, depth, etc..

```
function GENERAL-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
       if there are no candidates for expansion then return failure
       choose a leaf node for expansion according to strategy
       if the node contains a goal state then return the corresponding solution
       else expand the node and add the resulting nodes to the search tree
   end
```

Essence of search: which node to expand first?

$\longrightarrow$ search strategy

A strategy is defined by picking the *order of node expansion*

Types of Search

Uniformed : use only information available in problem definition

Heuristic : exploits some knowledge of the domain

Uninformed search strategies

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening search
6. Bidirectional search

Search strategies

Criteria for evaluating search:

1. Completeness: does it always find a solution if one exists?
2. Time complexity: number of nodes generated/expanded
3. Space complexity: maximum number of nodes in memory
4. Optimality: does it always find a least-cost solution?

Time/space complexity measured in terms of:

- b: maximum branching factor of the search tree
- d: depth of the least-cost solution
- m: maximum depth of the state space (may be ∞)

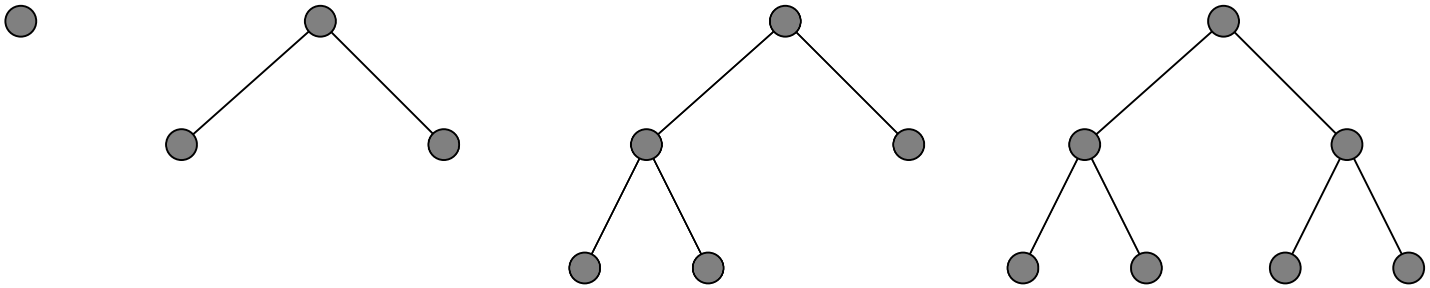
Breadth-first search (I)

→ Expand root node

→ Expand <u>*all*</u> children of root

→ Expand *each* child of root

→ Expand successors of each child of root, etc.



⟶ Expands nodes at depth d before nodes at depth $d+1$

⟶ Systematically considers all paths length 1, then length 2, etc.

⟶ Implement: put successors at end of queue

Breadth-first search (II)

$\longrightarrow$ One solution?

$\longrightarrow$ Many solutions? Finds shallowest goal first

1. Complete?
2. Optimal? (provided cost increases monotonically with depth)
3. Time? $O(b^d)$ $\begin{cases} \text{branching factor } b \\ \text{depth } d \end{cases}$
4. Space? same, $O(b^d)$, keeps every node in memory, big problem

Depth	Nodes	Time		Memory	
0	1	1	millisecond	100	bytes
2	111	.1	seconds	11	kilobytes
4	11,111	11	seconds	1	megabyte
6	10^6	18	minutes	111	megabytes
8	10^8	31	hours	11	gigabytes
10	10^{10}	128	days	1	terabyte
12	10^{12}	35	years	111	terabytes
14	10^{14}	3500	years	11,111	terabytes

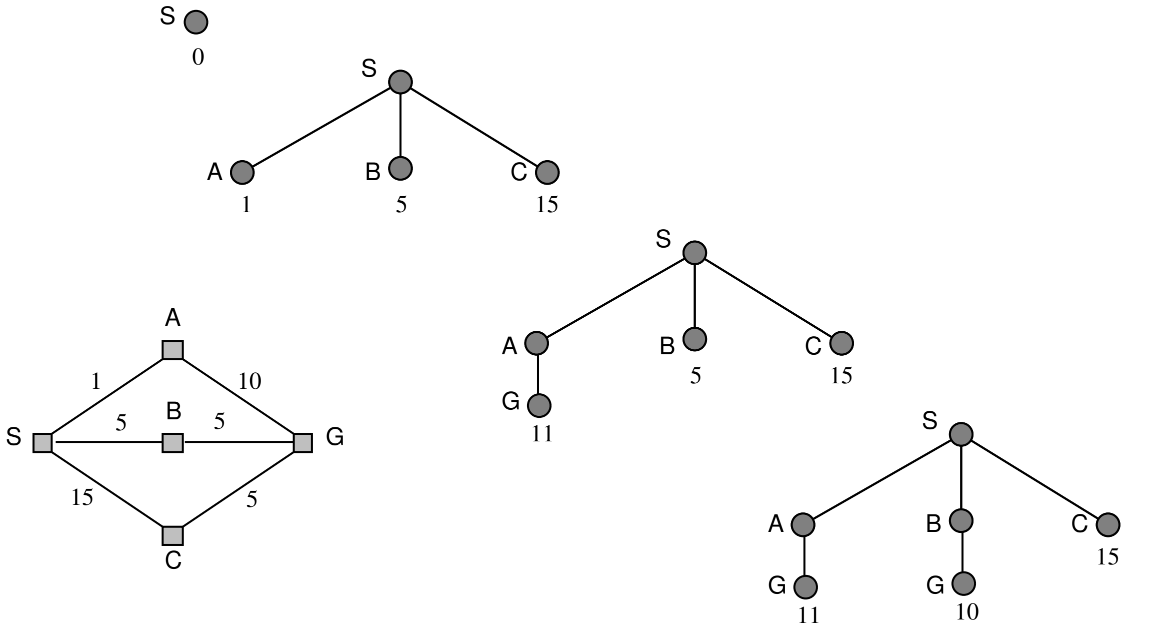
Uniform-cost search (I)

$\longrightarrow$ Breadth-first does not consider path cost $g(x)$

$\longrightarrow$ Uniform-cost expands first lowest-cost node on the fringe

$\longrightarrow$ Implement: sort queue in decreasing cost order

When $g(x) = \text{Depth}(x) \longrightarrow$ Breadth-first $\equiv$ Uniform-cost



(a) (b)

Uniform-cost search (II)

1. Complete?

   Yes, if cost $\geq \epsilon$

2. Optimal?

   If the cost is a monotonically increasing function

   When cost is added up along path, an operator's cost ....?

3. Time?

   \# of nodes with $g \leq$ cost of optimal solution

   worst-case $O(b^d)$

4. Space?

   \# of nodes with $g \leq$ cost of optimal solution

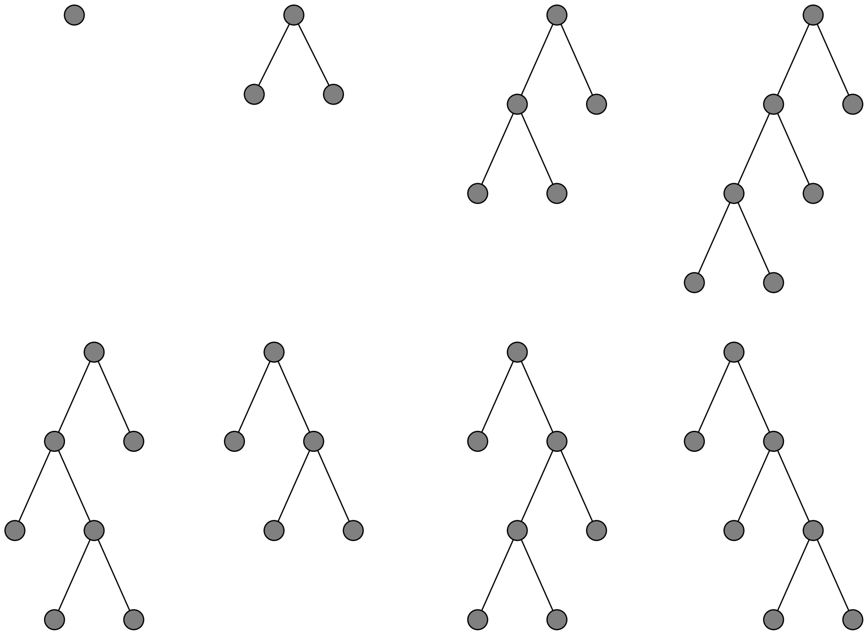
   worst-case $O(b^d)$

Depth-first search (I)

⟶ Expands nodes at deepest level in tree

⟶ When dead-end, goes back to shallower levels

⟶ Implement: put successors at end of queue (or, recursive calls)



⟶ Little memory: path and unexpanded nodes

For b: branching factor, m: maximum depth, space ....?

Depth-first search (II)

Time complexity:

We may need to expand all paths, $O(b^m)$

When there are many solutions, DFS may be quicker than BFS

When m is big, ∞ (deep, loops), .. troubles

$\longrightarrow$ Major drawback of DFS: going deep where there is no solution..

Properties:

1. Complete?
2. Optimal?
3. Time? Woow.
4. Space?

Depth-limited search (I)

⟶ DFS is going too deep, put a threshold on depth!
For instance, 20 cities on map for Romania, any node deeper than 19 is cycling. Don't expand deeper!

⟶ Implement: nodes at depth l have no successor

Properties:

1. Complete?
2. Optimal?
3. Time? (given l depth limit)
4. Space? (given l depth limit)

Problem: how to choose l?

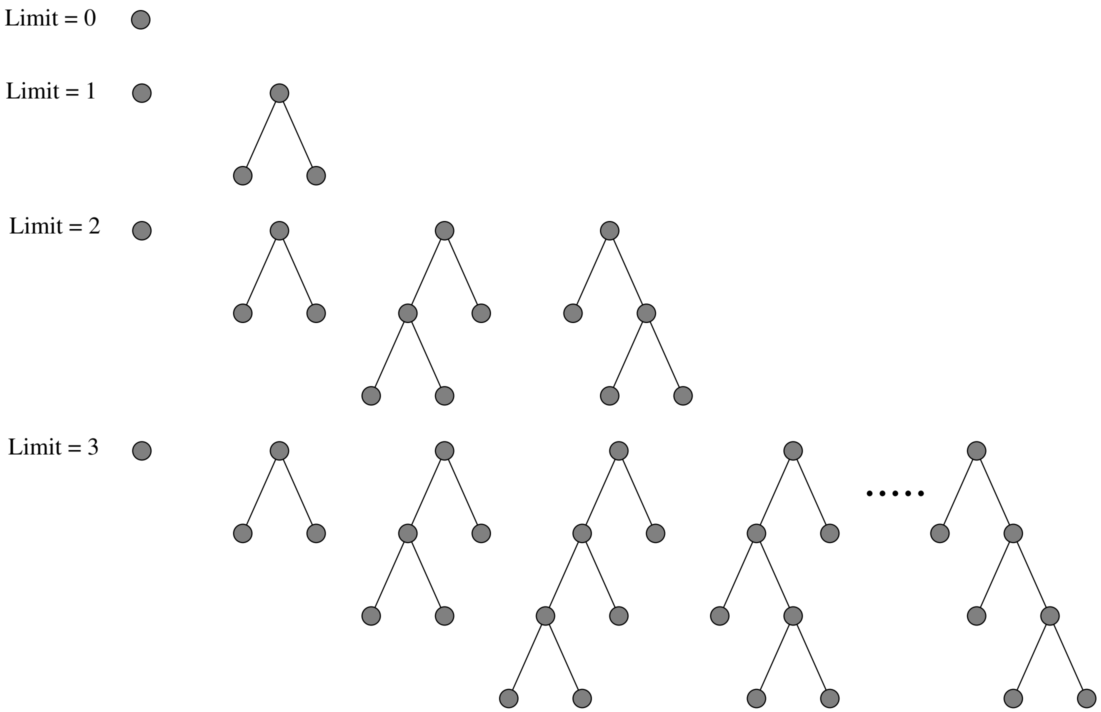
Iterative-deepening search (I)

→ DLS with depth = 0

→ DLS with depth = 1

→ DLS with depth = 2

→ DLS with depth = 3...



⟶ Combines benefits of DFS and BFS

Iterative-deepening search (II)

$\longrightarrow$ combines benefits of DFS and BFS

Properties:

1. Complete? like BFS
2. Optimal? like BFS (if step cost $= 1$)
3. Space? like DFS
4. Time? same

$\longrightarrow$ Some nodes are expanded several times, wasteful?

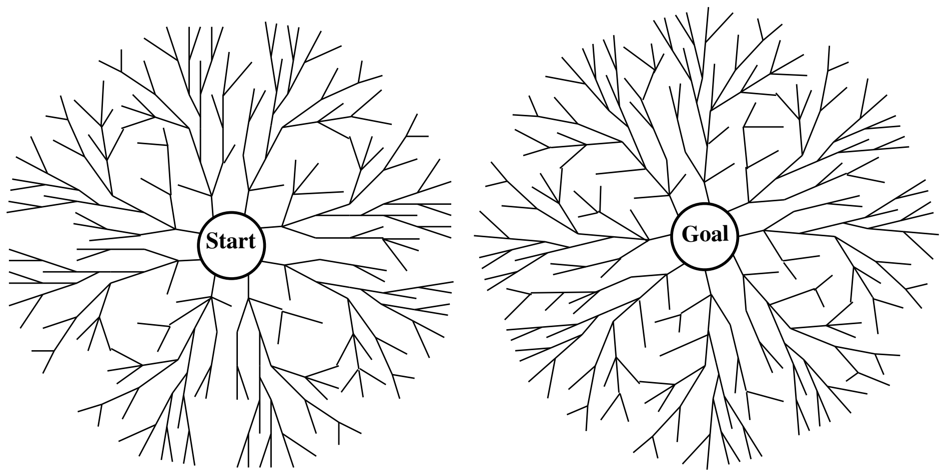
\# expansions in BFS: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$

\# in IDS: $(d+1).1 + (d).b + (d-1).b^2 + \ldots + 1.b^d = O(b^d)$

$\longrightarrow$ IDS is preferred when search space is large and depth unknown

Bidirectional search (I)

→ Given initial state and the goal state, start search from both ends and meet in the middle



→ Assume same b branching factor, $\exists$ solution at depth d, time: $O(2b^{d/2}) = O(b^{d/2})$

$b = 10, d = 6$, DFS= 1,111,111 nodes, BDS=2,222 nodes!

Bidirectional search (II)

In practice :—(

- Need to define predecessor operators to search backwards
  If operator are invertible, no problem
- What if $\exists$ many goals (set state)?
  do as for multiple-state search
- need to check the 2 fringes to see how they match
  need to check whether any node in one space appears in the other space (use hashing)
  need to keep all nodes in a half in memory $O(b^{d/2})$
- What kind of search in each half space?

Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

b branching factor

d solution depth

m maximum depth of tree

l depth limit