Sorting in Linear Time

Textbook, Chapter 9, Sections 9.2 and 9.3

CSCE310: Data Structures and Algorithms
`www.cse.unl.edu/~choueiry/S01-310/`
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$O(n \lg n)$:

- Mergesort, heapsort: worst-case
- quicksort: average-case

$\Omega(n \lg n)$:

- Mergesort, heapsort, quicksort

Interesting common property: sorted order is based *only* on comparisons between the input elements

→ **Comparison sorts algorithms**

We can prove that: *(Section 1.1)*
any comparison sort algorithm is in $\Omega(n \lg n)$

Non-comparison sort algorithms

- Counting sort: assumes something about input
  $O(n)$, stable
- Radix sort, $\Theta(dn + kd)$
  When d constant, $k = O(n) \Rightarrow$ linear time
- Bucket sort: assumes something about input
  $O(n)$

Counting sort

Assumes that each of the n input element is an integer in the range of 1 to k

When $k = O(n)$, counting sort is linear

Principle

- Determine for each input element x, the number of elements less than x
- Every element x can be placed directly in its position in output array

Example
If $\exists$ 17 elements less than x, x must be in position 18

Slight modification for same value cases

Counting sort

Input: Array $A[1 \ldots n], length[A] = n$

Output: Array $B[1 \ldots n]$

Temporary working storage: Array $C[1 \ldots 2]$

```
COUNTING-SORT(A, B, k)
 1  for i ← 1 to k
 2       do C[i] ← 0
 3  for j ← 1 to length[A]
 4       do C[A[j]] ← C[A[j]] + 1
 5  ▷ C[i] now contains the number of elements equal to i.
 6  for i ← 2 to k
 7       do C[i] ← C[i] + C[i − 1]
 8  ▷ C[i] now contains the number of elements less than or equal to i.
 9  for j ← length[A] downto 1
10       do B[C[A[j]]] ← A[j]
11           C[A[j]] ← C[A[j]] − 1
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Lines 1–2: initialization

Lines 3–4: inspect each element, get values of $C[i]$

$C[i]$: number of elements equal to i

Lines 6–7: number of elements $\leq i$ (a running sum of C)

Lines 9–11: $A[j]$ is place in correct position in B

Counting sort

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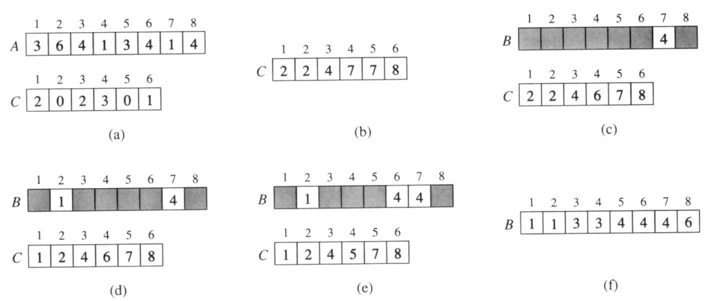
Lines 9–11: $A[j]$ is place in correct position in B

correct final position for $A[j]$ is $C[A[j]]$

Since some x may not be different,

need to decrement $C[A[j]]$ when placing an $A[j]$ into B



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Lines 1–2: $O(k)$

Lines 3–4: $O(n)$

Lines 6–7: $O(k)$

Lines 9–11: $O(n)$

Counting sort: $O(k+n)$

Usually, used with $k = O(n)$, this in $O(n)$

Counting sort: stable

Numbers with the same value appear in B in same order as in A

Important in presence of satellite data

Exercise: 9.2-1

Try again in 9.2-3

Radix sort

Given numbers of d-digit, Radix-sort:

1. Starts with the least significant digit first
2. Sorts the numbers according this digit
   using a stable sorting algorithm
3. Moves to the next least-significant digit
4. Repeats from 2, until last digit d
5. .. and the numbers are sorted!

Digit sorting must be stable

Radix sort is stable

Example: sort records by dates (years, months, and days)

General use: sort records keyed by multiple fields

Input: A, array of n elements,
each of d digits: 1 lowest-order digit, d highest-order digit

For $i \leftarrow 1$ **to** d
do use a stable sort to sort array A on digit i

329		720		720		329
457		355		329		355
657		436		436		436
839	⇒	457	⇒	839	⇒	457
436		657		355		657
720		329		457		720
355		839		657		839
		↑		↑		↑

Correctness: proof by induction on column being sorted

Running time: depends on intermediate sorting algorithm

Exercise: 9.3-1

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Running time: of Radixsort

When each digit is in the range of 1 to k (k not too large)

Use Counting sort

Each pass over n d-digit numbers is $\Theta(n + k)$

d-passes: $\Theta(dn + kd)$

When d constant and $k = O(n)$, Radixsort is linear!!

Unlike Quicksort and Insertionsort,
Countingsort does <u>not</u> sort in place

When space is at stake, use Quicksort

Non-comparison sort algorithms

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When $k = O(n)$, counting sort is linear

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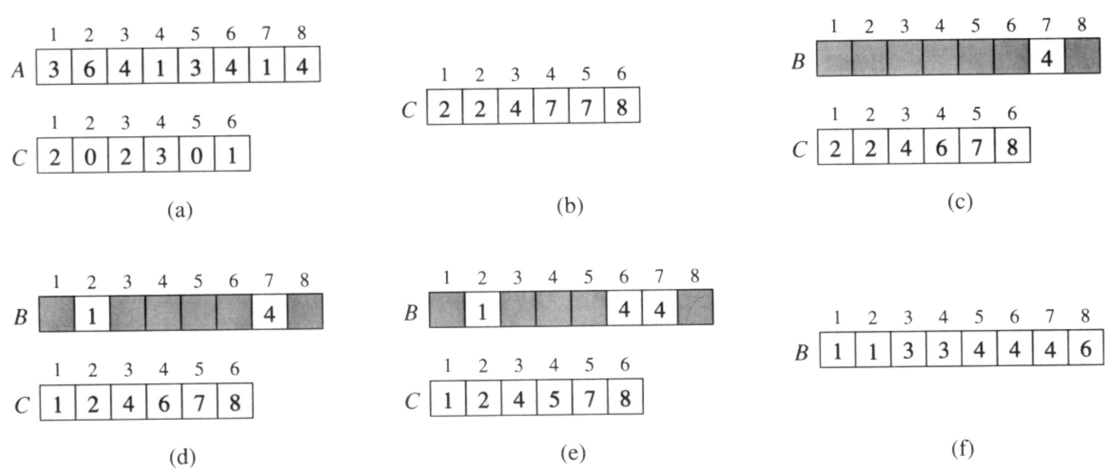

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