# Review of Recursion

Courtesy of Dr. Chuck Cusack

**CSCE310: Data Structures and Algorithms**

`www.cse.unl.edu/~choueiry/S01-310/`


Berthe Y. Choueiry (Shu-we-ri)

Ferguson Hall, Room 104

`choueiry@cse.unl.edu`, Tel: (402)472-5444

# Recursion

- A subroutine/function that calls itself is called *recursive*
- Warning: A subroutine/function that simply calls itself would result in infinite recursion
- Thus, when using recursion, one must ensure that, at some point, the subroutine/function terminates without calling itself

Two (2) fundamental rules of recursion:

1. **Base case(s)**, solved without recursion.
   A.k.a Stopping case, Terminating condition.
2. **Making progress**. Recursive calls must always be to a case that makes progress toward a base case.
   A.k.a inductive case(s) always progress toward a base case

Two other rules (later)

```
RecursiveFactorial(n)
  IF (n =< 1) THEN RETURN 1
  RETURN n * Factorial(n-1)
```

What ensures that the function `RecursiveFactorial` terminates?

# Where is Recursion Seen/Used?

In problems that can be solved by combining solutions of <u>smaller</u> instances of the given problem

**Toy problems:** Russian Matryoshka (nested dolls), Tower of Hanoi

**CS examples:** binary search, mergesort, $n!$, Fibonacci numbers, divide-&-conquer algorithms

# Iteration vs. Recursion: examples

## Iterative count down:

```
CountDown (n)
WHILE i > 0
      DO Print (n)
         n <-- (n - 1)
```

## Recursive count down: *Incorrect*

```
CountDown (n)
  Print(n)
  CountDown(n-1)
```

What is wrong?

## Recursive count down: *Fixed*

```
CountDown (n)
  IF n > 0
     THEN Print(n)
          CountDown(n-1)
```

# Recursion and Memory

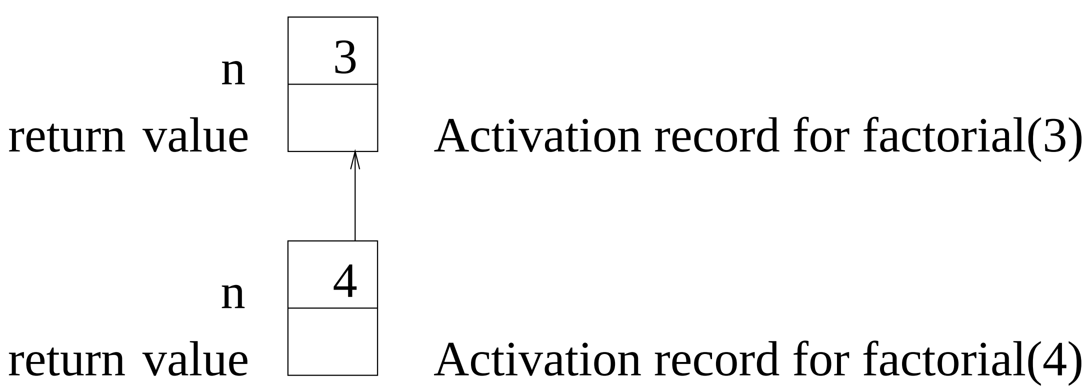
- Each function call generates a function instance
- An instance of a function contains
  - memory for each parameter (input)
  - memory for each local variable
  - memory for the return value

  This chunk of memory is referred to as **activation record**
- A recursive function that calls itself $n$ times must allocate $n$ activation records
- Usually, an iterative implementation will require on the order of one activation record, plus a constant amount of space
- Space: reason recursion for avoiding recursion
- Recursion simplifies code readability (good compilers remove recursion whenever possible)

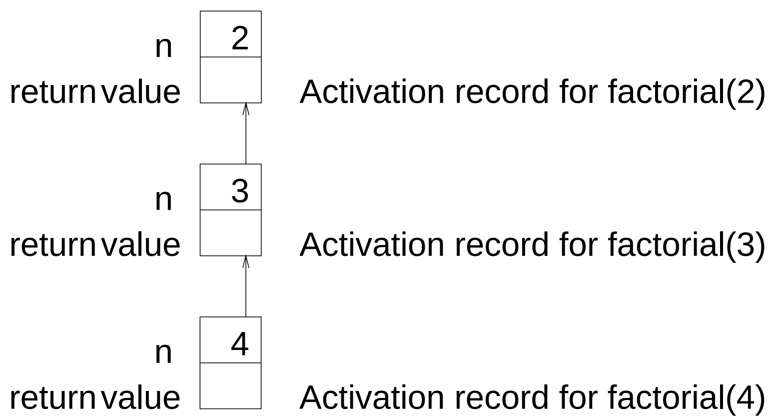
## Example:

- `RecursiveFactorial(4)`

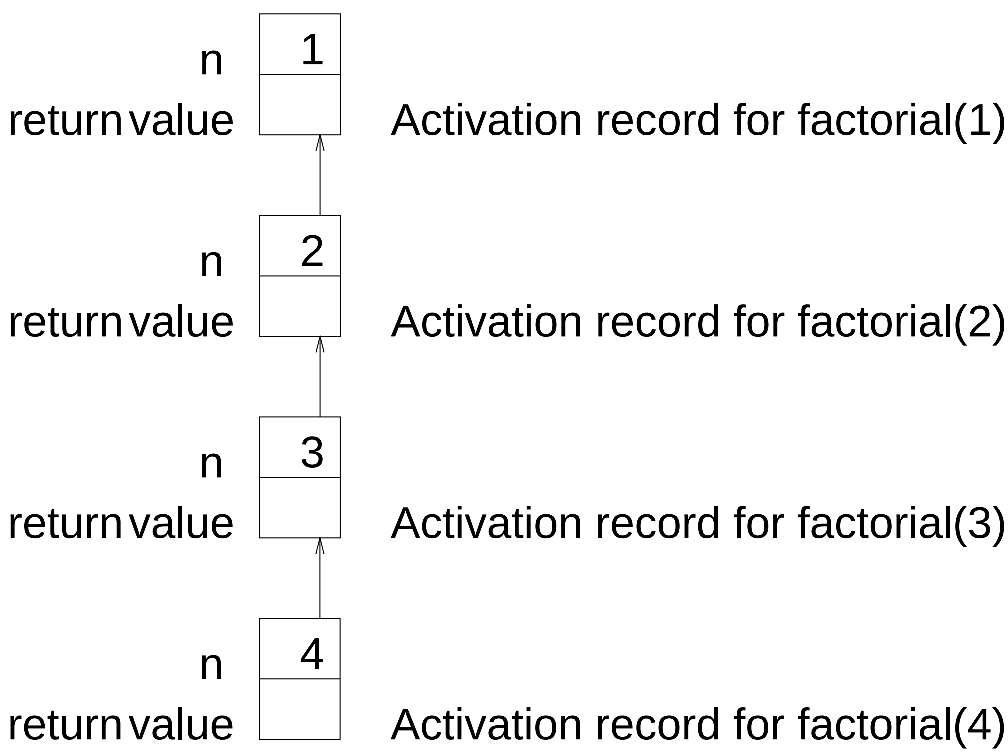

- `RecursiveFactorial(4)`: call to `RecursiveFactorial(3)`



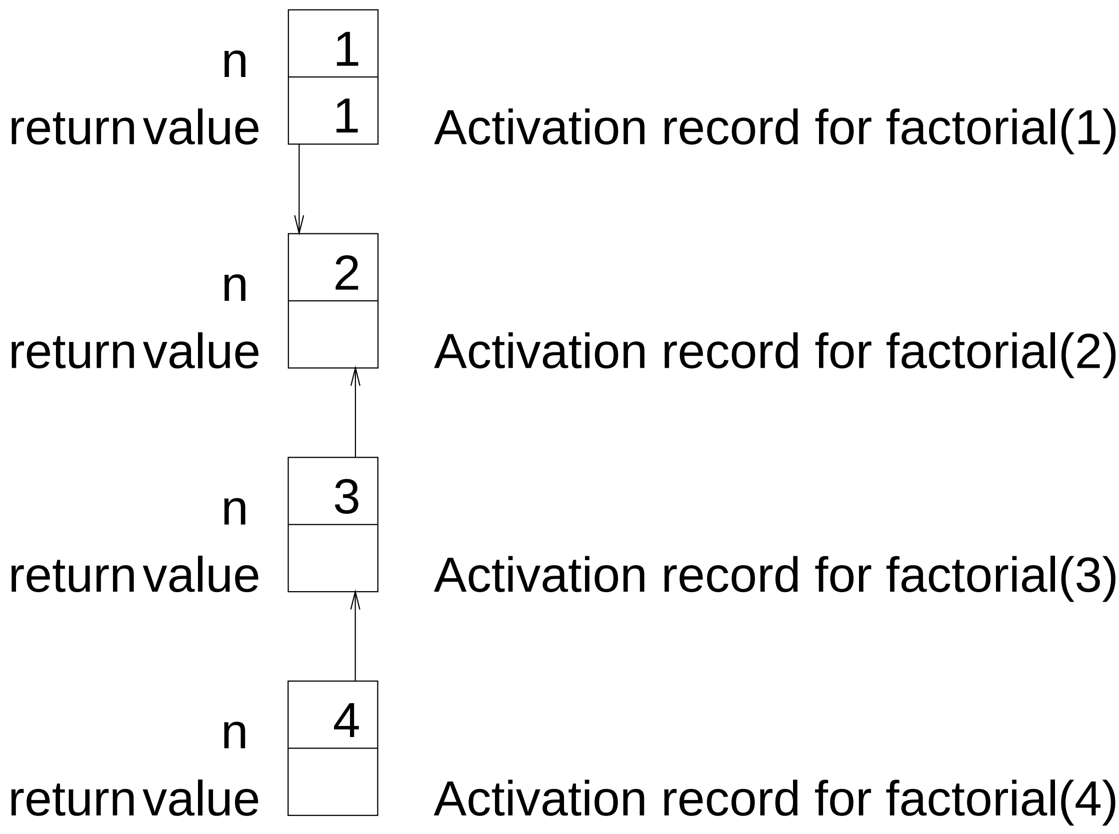
- `RecursiveFactorial(4)`: call to `RecursiveFactorial(2)`




- `RecursiveFactorial(4)`: call to `RecursiveFactorial(1)`

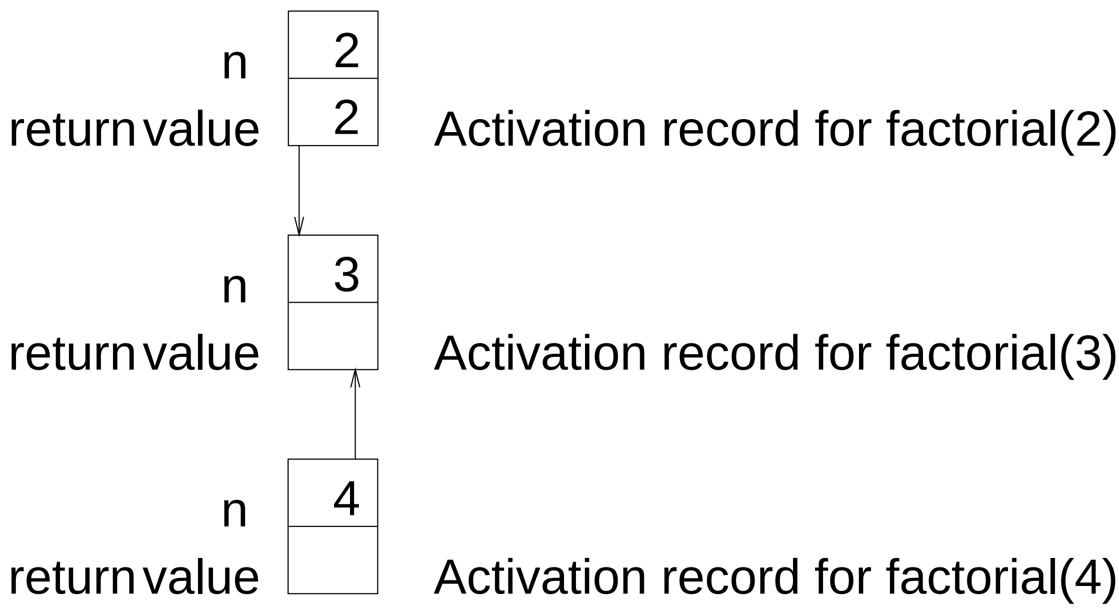


- `RecursiveFactorial(4)`:
  `RecursiveFactorial(1)` is the base case, so it returns '1'.

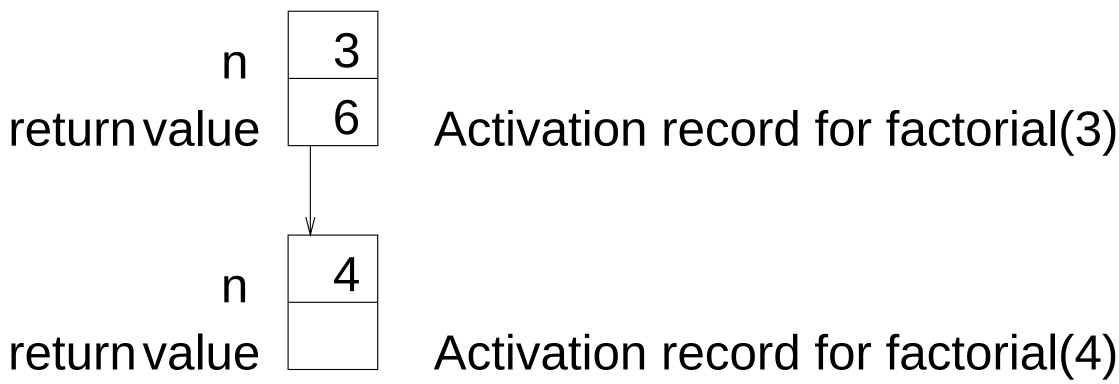


- `RecursiveFactorial(4)`:

  `RecursiveFactorial(2)` now returns ‘2’.



- `RecursiveFactorial(4)`:
  `RecursiveFactorial(3)` now returns '6'.




- `RecursiveFactorial(4)`: returns '24'. This was the original function call, so the execution is finished.

# The Run-Time Stack

- In order to support recursive function calls, the run-time system treats memory as a stack of activation records
- `RecursiveFactorial(n)` requires the allocation of $n$ activation records on the stack

```
RecursiveFactorial(n)
IF (n =< 1) THEN 1
RETURN n * RecursiveFactorial(n-1)
```

- What if we have infinite recursion:

```
InfiniteRecursion (n)
IF (n=0) THEN 1
         ELSE InfiniteRecursion(n)
```

  The value of $n$ never reaches zero, so the function is called, and records are pushed onto the stack, until the system runs out of memory.

- Even if our recursion is not infinite, system may run out of memory: the recursion may be too deep, and since memory is finite

## Recursion vs. iteration

Recursive functions can be translated to functions that use loops.

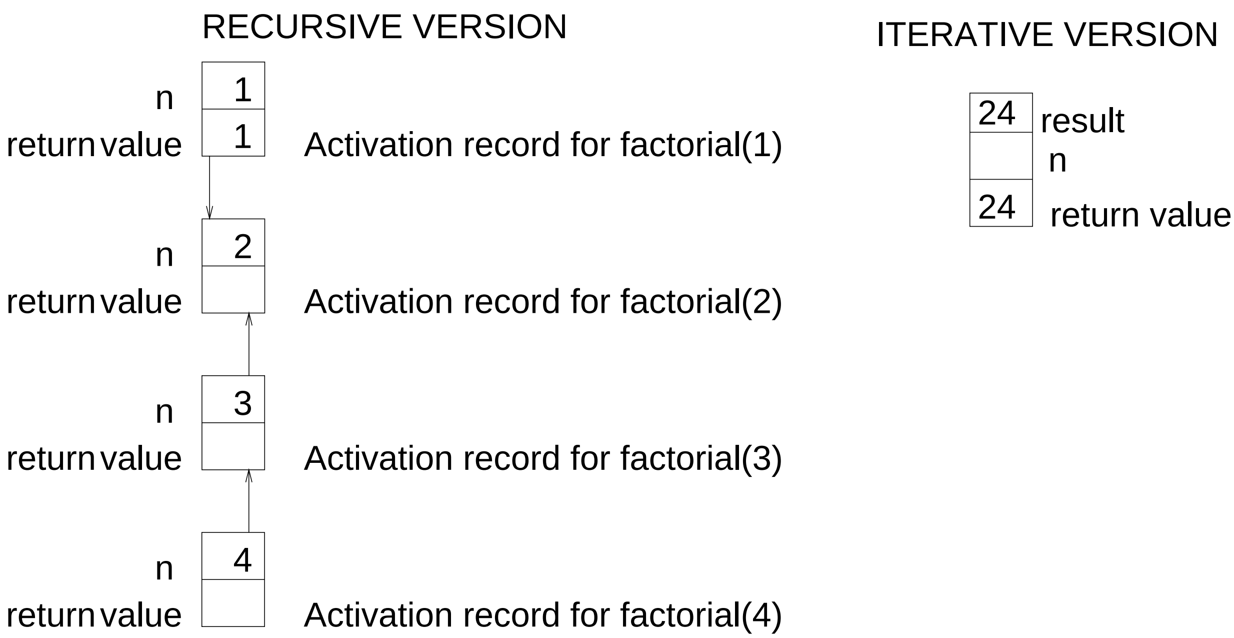
- **Recursive:**

```
RecursiveFactorial(n)
   IF (n =< 1) THEN 1
   n * Factorial(n-1)
```

- **Iterative:**

```
IterativeFactorial(n)
result <-- 1
WHILE (n >= 1)
      result <-- result * n
      n <-- n - 1
RETURN  result
```

# Memory Usage?




For input $n$:

**Recursive implementation** needs to allocate $2n$ integers

**Iterative implementation** needs only 3

## **Recursion**: Disadvantages

Each time one function calls another, the computer's operating system must:

- Record how to re-start the calling function later on,
- Pass the parameters from the calling function to the called function (often by pushing the parameters onto a stack controlled by the system)
- Set up space for the called function's local variables
- Record where the calling function's local variables are stored

Doing all this requires time and memory.

# Common recursion errors

- Forgetting or having incomplete base cases

```
Sum1toN(N)
IF (N == 0)
   THEN RETURN 0
   ELSE RETURN (N + Sum1toN(N-1))
```

- Getting things backwards

```
CountHow(n)
IF n > 0
   THEN Print(n)
        CountHow(n-1)

CountGuess(n)
IF n > 0
   THEN CountGuess(n-1)
        Print(n)
```

Two (2) fundamental rules of recursion:

1. Base case(s)
2. Making progress

Two other rules:

1. **Design rule**. Assume all recursive calls work.
   Don't do the bookkeeping yourself.
2. **Compound interest rule**.
   Don't duplicate work by solving the same instance of a problem in separate recursive calls.

**Recursion**: Conclusion

- A recursive function is one that invokes another instance of itself
- Recursion is an alternative to iteration
- Recursion often provides a more elegant solution than iteration
- Each instance of a function has its own set of local variables and parameters
- Recursive solutions are often less efficient, in terms of time and space, than an iterative solution.
- Some problems are difficult to solve without recursion. Particularly when the problem is recursive in its definition, example Tower of Hanoi.

Exercise 11.4-2 (not tested)

```
Print-node (node)
IF (left-child[node]=nil) AND (right-sibling[node]=nil)
   THEN print(key[node])
   ELSE IF right-sibling[node]=nil
           THEN print(key[node])
                print-node (left-child[node])
           ELSE IF left-child[node] = nil
                   THEN print(key[node])
                        print-node (right-sibling[node])
                   ELSE print(key[node])
                        print-node (left-child[node])
                        print-node (right-sibling[node])
```

# Common functions

Floors and ceilings:

- The floor of $x$: $\lfloor x \rfloor$
- The ceiling of $x$: $\lceil x \rceil$
- $\forall\, n \in \mathbb{N}$, $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$

Logarithms:

- Binary logarithm: $\lg n = \log_2 n$
- Natural logarithm: $\ln n = \log_e n$
- $\log_a n = \frac{\ln n}{\ln a}$ and $\log n = \frac{\ln n}{\ln 10}$
- $\ln e = 1$, $\ln 1 = 0$, $\log_k 1 = 0$, $\log_k k = 1$
- Exponentiation: $\lg^k n = (\lg n)^k$
- Composition: $\lg \lg n = \lg(\lg n)$
- $a = b^{\log_b a}$
- $\log_b a^n = n \log_b a$
- $a^{\log_b n} = n^{\log_b a}$

Sum of finite series:

- Arithmetic: $S_n = \frac{n(t_1 + t_n)}{2}$
- Geometric: $S_n = t_1 \frac{r^n - 1}{r - 1} = \frac{t_n r - t_1}{r - 1}$