Heapsort (II)

Textbook, Chapter 7

CSCE310: Data Structures and Algorithms

`www.cse.unl.edu/~choueiry/S01-310/`

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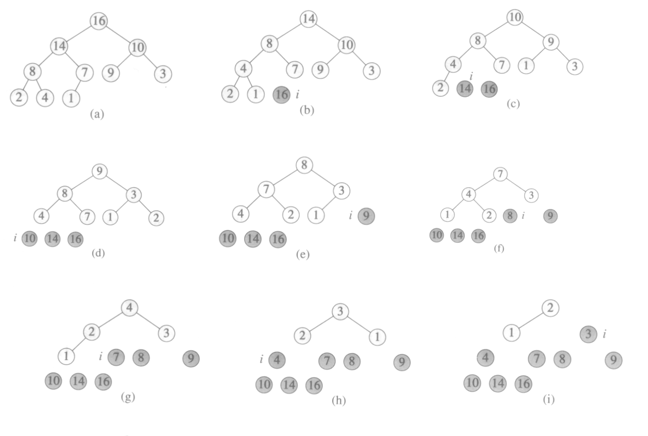
Heapsort: Principle and algorithm

Input: Array $A[1 \dots n]$, where $n = length[A]$

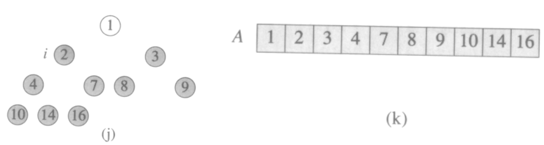
1. Build a heap out of the array
   the maximum element will be in $A[1]$, it should be last!!
2. Exchange $A[1]$ with $A[n]$
3. Reset heap size to $(n-1)$
4. Heapify $A[1 \dots (n-1)]$, repeat... down to a heap of size 2

```
HEAPSORT(A)
BUILD-HEAP(A)
for i ← length[A] downto 2
    do exchange A[1] ↔ A[i]
       heap-size[A] ← heap-size[A] − 1
       HEAPIFY(A, 1)
```


Heapsort: Example $A = \langle 4, 1, 3, 2, 16, 9, 10, 14, 8, 7 \rangle$



Heapsort: Example $A = \langle 4, 1, 3, 2, 16, 9, 10, 14, 8, 7 \rangle$ *(cont')*



Heapsort: Complexity

For an array A of size n

- `Build-Heap` is called once, its complexity id $O(n)$
- `Heapify` is called $n-1$ times, its complexity is $O(\lg n)$
- Complexity of `Heapsort` is $O(n+(n-1)\lg n) = O(n \lg n)$

Heapsort: Utility

- `Heapsort` is an excellent algorithm, however `Quicksort` is better in practice
- Heap data structure has enormous utility
  e.g., efficient priority queue

Priority queue as a data structure

- Data structure for maintaining a set S of elements, each with a value called key
- Supports operations:
  `Insert`(S, x) (i.e., $S \leftarrow S \cup \{x\}$)
  `Maximum`(S) returns element of S with largest key
  `Extract-Max`(S) removes and returns the element of S with largest key

Priority queue: Applications

1. Job scheduling on a shared resource (e.g., computer)
   Priority queue keeps track of jobs and their relative importance.
   `Insert` allows us to add new jobs at any time
   `Extract-Max` selects highest priority jobs when current job is finished/interrupted
2. Event-driven simulator
   Queue of events to be simulated, each has occurrence time → key value
   Occurrence of event causes other events to be simulated in future → `Insert` `Extract-Min` and `Minimum` instead of `Extract-Max` and `Maximum`

Implementation → a heap

`Heap-Extract-Max`(A)

If heap-size$[A] < 1$
  then error "heap underflow"
max $\leftarrow A[1]$
$A[1] \leftarrow A$[heap-size$[A]$]
heap-size$[A] \leftarrow$ heap-size$[A] - 1$
`Heapify`$(A, 1)$
return max

→ Returns $A[1]$
→ Places the last element of the heap in $A[1]$
→ Decrements size of heap
→ Heapifies the array
→ Running time is in $O(\lg n)$ (constant effort and 1 heapify)

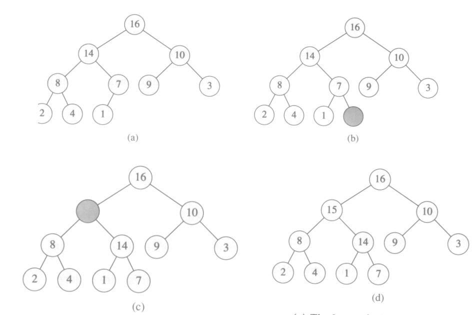
`Heap-Insert`(A, key)

heap-size$[A] \leftarrow$ heap-size$[A] + 1$
$i \leftarrow$ heap-size$[A]$
while $i > 1$ and $A[Parent(i)] < key$
    do $A[i] \leftarrow A[Parent(i)]$
        $i \leftarrow Parent(i)$
$A[i] \leftarrow key$

$\rightarrow$ First expands the heap by adding 1 new leaf
$\rightarrow$ Then traverses path from new leaf to root to find a proper place for the new element

Heap-Insert(A, key): Example

$key = 15$



Running time is $O(\lg n)$, since path from new leaf to root has length $\lg n$

Heap as a priority queue

For any set S of n elements:

- `Heap-Maximum`(A) is in $\Theta(1)$
- `Heap-Extract-Max`(A) is in $O(\lg n)$
- `Heap-Insert`(A, key) is in $O(\lg n)$

Bubble-sort

Notes of Dr. Cusack

- Go through the list in order, swapping two elements if their keys are out of order
- Repeat until no swaps are performed. The list is sorted
- n passes suffice
- Similar to Insertion-sort, but has lots of swaps

Bubble-Sort (A)
For $i \rightarrow n-1$ downto 1
    For $j \rightarrow 1$ to i
        When $A[j-1] > A[j]$, $A[j-1] \leftrightarrow A[j]$

Name	Upper bound	Lower bound	Tight bound
Selection-sort	$O(n^2)$	$\Omega(n)$	
Insertion-sort	$O(n^2)$	$\Omega(n)$	
Bubble-sort	$O(n^2)$		
Merge-sort			$\Theta(n \lg n)$
Heap-sort	$O(n \lg n)$		
Quick-sort	$O(n^2)$	$\Omega(n \lg n)$	