

Title: First-Order Logic

AIMA: Chapter 8 (Sections 8.1 and 8.2)

Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence

CSCE 476-876, Fall 2023

URL: www.cse.unl.edu/~choueiry/F23-476-876

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Outline

- First-order logic:
 - basic elements
 - syntax
 - semantics
- Examples

Pros and cons of propositional logic

- Propositional logic is declarative: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
(unlike most data structures and databases)
- Propositional logic is compositional:
meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent
(unlike natural language, where meaning depends on context)
- but...
Propositional logic has very limited expressive power
E.g., cannot say “pits cause breezes in adjacent squares”
except by writing one sentence for each square

Propositional Logic

- is simple
- illustrates important points:
model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness:
→ In PL, world contains facts

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)

First Order Logic

- FOL provides more "primitives" to express knowledge:
- objects (identity & properties)
 - relations among objects (including functions)

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Objects: people, houses, numbers, Einstein, Huskers, event, ..

Properties: smart, nice, large, intelligent, loved, occurred, ..

Relations: brother-of, bigger-than, part-of, occurred-after, ..

Functions: father-of, best-friend, double-of, ..

Examples: (objects? function? relation? property?)

— one plus two equals four [sic]

— squares neighboring the wumpus are smelly

Logic

Attracts: mathematicians, philosophers and AI people

Advantages:

- allows to represent the world and reason about it
- expresses anything that can be programmed

Non-committal to:

- symbols could be objects or relations
(*e.g.*, King(Gustave), King(Sweden, Gustave), Merciless(King))
- classes, categories, time, events, uncertainty

.. but amenable to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

→ Some people think FOL **is** the language of AI
true/false? donno :—(but it will remain around for some time..

Types of logic

Logics are characterized by what they commit to as “primitives”

Ontological commitment :

what exists—facts? objects? time? beliefs?

Epistemological commitment :

what states of knowledge?

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0...1
Fuzzy logic	degree of truth	degree of belief 0...1

Higher-Order Logic: views relations and functions of FOL as objects

Syntax of FOL: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: x , y , etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation) Father-of, Square-root, LeftLeg, etc.
- Quantifiers \forall , \exists
- Connectives: \wedge , \vee , \neg , \Rightarrow , \Leftrightarrow ,
- (Sometimes) equality =

Predicates and functions can have any arity (number of arguments)

Basic elements in FOL (i.e., the grammar)

In FOL,

- Terms
- Sentences:
 - atomic sentences
 - complex sentences
- Quantifiers:
 - Universal quantifier
 - Existential quantifier

Term

logical expression that refers to an object

- built with: constant symbols, variables, function symbols

$$\text{Term} = \textit{function}(\textit{term}_1, \dots, \textit{term}_n)$$

or constant or variable

- **ground term**: term with no variable

Atomic sentences

state facts

built with terms and predicate symbols

Atomic sentence = $predicate(term_1, \dots, term_n)$
or $term_1 = term_2$

Examples:

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))

Complex Sentences

built with atomic sentences and logical connectives

$$\neg S$$

$$S_1 \wedge S_2$$

$$S_1 \vee S_2$$

$$S_1 \Rightarrow S_2$$

$$S_1 \Leftrightarrow S_2$$

Examples:

$\text{Sibling}(\text{KingJohn}, \text{Richard}) \Rightarrow \text{Sibling}(\text{Richard}, \text{KingJohn})$

$$\gt(1, 2) \vee \leq(1, 2)$$

$$\gt(1, 2) \wedge \neg \gt(1, 2)$$

Truth in first-order logic: Semantic

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for

constant symbols → objects

predicate symbols → relations

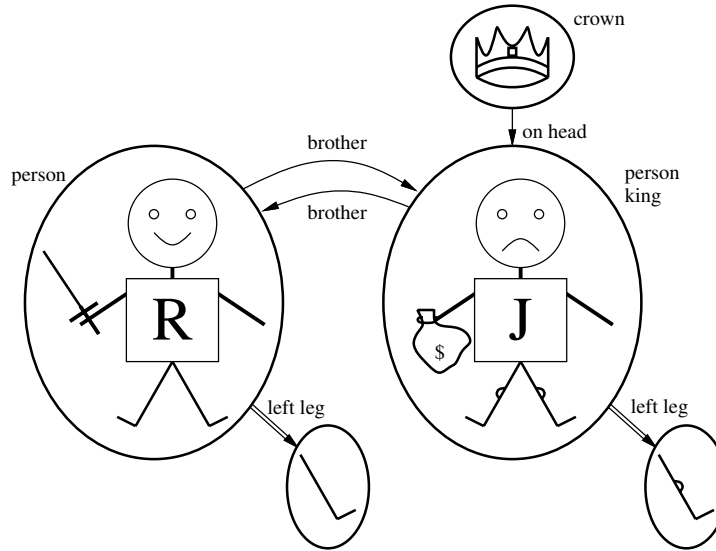
function symbols → functional relations

An atomic sentence $predicate(term_1, \dots, term_n)$ is true

iff the objects referred to by $term_1, \dots, term_n$

are in the relation referred to by *predicate*

Model in FOL: example



The domain of a model is the set of objects it contains:
five objects

Intended interpretation: Richard refers Richard the Lion Heart,
John refers to Evil King John, Brother refers to brotherhood
relation, etc.

Models for FOL: Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞

For each k -ary predicate P_k in the vocabulary

For each possible k -ary relation on n objects

For each constant symbol C in the vocabulary

For each choice of referent for C from n objects ...

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded

→ Checking entailment by enumerating is not an option

Model: Compare...

- Logic: A model of a sentence is a world in which a sentence is true under a particular interpretation.
- Propositional Logic: A model of a sentence is a mapping of the propositions directly to truth or falsehood such that the sentence holds.
- FOL: A model of a sentence in FOL is
 1. a set of objects and
 2. an interpretation, which is a mapping of:
 - Constant symbols to objects
 - function symbols to functions on those objects
 - predicate symbols to relations over those objectswhere the sentence holds.

Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things

Universal quantification

\forall *<variables>* *<sentence>*

Example: all dogs like bones $\forall x \text{Dog}(x) \Rightarrow \text{LikeBones}(x)$

$x = \text{Indy}$ is a dog

$x = \text{Indiana Jones}$ is a person

$\forall x P$ is equivalent to the conjunction of instantiations of P

$\text{Dog}(\text{Indy}) \Rightarrow \text{LikeBones}(\text{Indy})$

$\wedge \text{Dog}(\text{Rebel}) \Rightarrow \text{LikeBones}(\text{Rebel})$

$\wedge \text{Dog}(\text{KingJohn}) \Rightarrow \text{LikeBones}(\text{KingJohn})$

$\wedge \dots$

Typically: \Rightarrow is the main connective with \forall

Common mistake: using \wedge as the main connective with \forall

Example: $\forall x \text{Dog}(x) \wedge \text{LikeBones}(x)$

all objects in the world are dogs, and all like bones

Existential quantification

$\exists \langle \text{variables} \rangle \langle \text{sentence} \rangle$

Example: some student will talk at the TechFair

$\exists x \text{Student}(x) \wedge \text{TalksAtTechFair}(x)$

Pat, Leslie, Chris are students

$\exists x P$ is equivalent to the disjunction of instantiations of P

$\text{Student}(\text{Pat}) \wedge \text{TalksAtTechFair}(\text{Pat})$

$\vee \text{Student}(\text{Leslie}) \wedge \text{TalksAtTechFair}(\text{Leslie})$

$\vee \text{Student}(\text{Chris}) \wedge \text{TalksAtTechFair}(\text{Chris})$

$\vee \dots$

Typically: \wedge is the main connective with \exists

Common mistake: using \Rightarrow as the main connective with \exists

$\exists x \text{Student}(x) \Rightarrow \text{TalksAtTechFair}(x)$

is true if there is anyone who is not Student

Properties of quantifiers (I)

$\forall x \forall y$ is the same as $\forall y \forall x$

$\exists x \exists y$ is the same as $\exists y \exists x$

$\exists x \forall y$ is not the same as $\forall y \exists x$

$\exists x \forall y \text{ Loves}(x, y)$

“There is a person who loves everyone in the world”

$\forall y \exists x \text{ Loves}(x, y)$

“Everyone in the world is loved by at least one person”

Quantifier duality: each can be expressed using the other

$\forall x \text{ Likes}(x, \text{IceCream}) \quad \neg \exists x \neg \text{Likes}(x, \text{IceCream})$

$\exists x \text{ Likes}(x, \text{Broccoli}) \quad \neg \forall x \neg \text{Likes}(x, \text{Broccoli})$

Parsimony principal: \forall , \neg , and \Rightarrow are sufficient

Properties of quantifiers (II)

Nested quantifier:

$\forall x(\exists y(P(x, y)):$

every object in the world has a particular property, which is the property to be related to some object by the relation P

$\exists x(\forall y(P(x, y)):$

there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

Lexical scoping: $\forall x[Cat(x) \vee \exists x Brother(Richard, x)]$

Well-formed formulas (WFF): (kind of correct spelling)

every variable must be introduced by a quantifier

$\forall xP(y)$ is not a WFF

Examples

Brothers are siblings

.

“Sibling” is symmetric

.

One’s mother is one’s female parent

.

A first cousin is a child of a parent’s sibling

Examples

.

$$\forall x, y \textit{ Brother}(x, y) \Rightarrow \textit{ Sibling}(x, y)$$

.

$$\forall x, y \textit{ Sibling}(x, y) \Rightarrow \textit{ Sibling}(y, x)$$

.

$$\forall x, y \textit{ Mother}(x, y) \Leftrightarrow (\textit{ Female}(x) \wedge \textit{ Parent}(x, y))$$

.

$$\forall x, y \textit{ FirstCousin}(x, y) \Leftrightarrow$$

$$\exists a, b \textit{ Parent}(a, x) \wedge \textit{ Sibling}(a, b) \wedge \textit{ Parent}(b, y)$$

Tricky example

Someone is loved by everyone

$$\exists x \forall y \text{ Loves}(y, x)$$

Someone with red-hair is loved by everyone

$$\exists x \forall y \text{ Redhair}(x) \wedge \text{ Loves}(y, x)$$

Alternatively:

$$\exists x \text{ Person}(x) \wedge \text{ Redhair}(x) \wedge (\forall y \text{ Person}(y) \Rightarrow \text{ Loves}(y, x))$$

Equality

$term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

Example: $Father(John) = Henry$

Equality is useful to distinguish two objects. Examples:

- Definition of (full) *Sibling* in terms of *Parent*.

$$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow [\exists m, f \neg(m = f) \wedge \text{Parent}(m, x) \wedge \text{Parent}(f, x) \wedge \text{Parent}(m, y) \wedge \text{Parent}(f, y)]$$

- Spot has at least two sisters: ...

AIMA, Exercise 8.4. Write: “All Germans speak the same languages,” where $Speaks(x, l)$ means that person x speaks language l .

Knowledge representation (KR)

Domain: a section of the world about which we wish to express some knowledge

Example: Family relations (kinship):

- Objects: people
- Properties: gender, married, divorced, single, widowed
- Relations: parenthood, brotherhood, marriage..

Unary predicates: Male, Female

Binary relations: Parent, Sibling, Brother, Child, etc.

Functions: Mother, Father

$$\forall m, c, \text{Mother}(c) = m \Leftrightarrow \text{Female}(m) \wedge \text{Parent}(m, c)$$

In Logic (informally)

- Basic facts: axioms (definitions)
- Derived facts: theorems

Independent axiom

an axiom that cannot be derived from the rest

→ Goal of mathematicians: find the minimal set of independent axioms

In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$

$Ask(KB, \exists a Action(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: *Yes*, $\{a/Shoot\}$ \leftarrow substitution (binding list)

Given a sentence S and a substitution σ ,

$S\sigma$ denotes the result of plugging σ into S ; e.g.,

$S = Smarter(x, y)$

$\sigma = \{x/Hillary, y/Bill\}$

$S\sigma = Smarter(Hillary, Bill)$

$Ask(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Prepare for next lecture: AIMA, Exercise 8.24, page 319

$\text{Takes}(x, c, s)$: student x takes course c in semester s

$\text{Passes}(x, c, s)$: student x passes course c in semester s

$\text{Score}(x, c, s)$: the score obtained by student x in course c in semester s

$x > y$: x is greater than y

F and G : specific French and Greek courses

$\text{Buys}(x, y, z)$: x buys y from z

$\text{Sells}(x, y, z)$: x sells y from z

$\text{Shaves}(x, y)$: person x shaves person y

$\text{Born}(x, c)$: person x is born in country c

$\text{Parent}(x, y)$: person x is parent of person y

$\text{Citizen}(x, c, r)$: person x is citizen of country c for reason r

$\text{Resident}(x, c)$: person x is resident of country c of person y

$\text{Fools}(x, y, t)$: person x fools person y at time t

$\text{Student}(x)$, $\text{Person}(x)$, $\text{Man}(x)$, $\text{Barber}(x)$, $\text{Expensive}(x)$, $\text{Agent}(x)$,

$\text{Insured}(x)$, $\text{Smart}(x)$, $\text{Politician}(x)$,

AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986

head of AI at AT&T Labs-Research

Program co-chair of AAAI-2000

Professor at University of Washington, Seattle

Founding Director of Institute for Data Science and Professor at University of Rochester