CSCE476/876 **Fall 2023**

Homework 7

Assigned on: Friday, November 10, 2023.

Due: Friday, November 17, 2023.

Points: 120 points + up to 20 bonus

Exercises: AIMA exercises are available online: `https://aimacode.github.io/aima-exercises`

Contents

Alert: If you submit your homework handwritten, it must be *absolutely neat* or it *will not* be corrected. If you type your homework (preferable), submit using webhandin.

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1 SAT Modeling (15 Points)

For each of the following scenarios, write a CNF formula to describe the scenario and complete the following four steps:

1. First state the propositions and what they represent.
2. State the sentence.
3. Explain the meaning of the clauses.
4. Is the sentence satisfiable? Explain why or why not.

1.1 Scenario A (5 Points)

Write a CNF formula to model the following scenario and complete the four steps from above:

1. There are four choices of desserts: ice cream, fruit bowl, cake, pie.
2. Exactly one dessert must be selected (i.e., one and only one).

1.2 Scenario B (5 Points)

Write a CNF formula to model the following scenario and complete the four steps from above:

1. Damon, Enrique, and Lois need to complete a paper and a presentation for a class.
2. To complete each task, they need to select a day to meet during the week (Mon, Tue, Wed, Thu, Fri).
3. Damon cannot meet on Monday. Further, he wants to complete the paper before the presentation and not both on the same day.
4. Enrique can meet any day but cannot meet on two consecutive days.
5. Lois wants to complete the presentation on or before Wednesday.

1.3 Scenario C (5 Points)

Write a CNF formula to model the following scenario and complete the four steps from above:

1. The four states (NE, IA, KS, MO) on the map shown in Figure 1 must be colored using three colors: red, green, and blue.

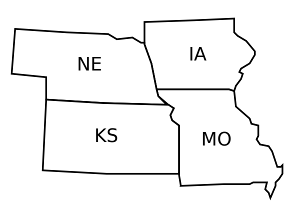

Figure 1: Four states (NE, IA, KS, MO)

2. Each state must be colored with exactly one color.
3. Adjacent states (i.e., states sharing a border line) cannot have the same color.

2 Chapter 7, Exercise 1, Source: AIMA online site. (16 points)

Suppose the agent has progressed to the point shown in Figure 7.4(a), Page 213, having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (You should find 32 of them.) Mark the worlds in which the KB is true and those in which each of the following sentences is true:

$$\alpha_2 = \text{"There is no pit in [2,2]."}$$

$$\alpha_3 = \text{"There is a wumpus in [1,3]."}$$

Hence show that $KB \models \alpha_2$ and $KB \models \alpha_3$.

3 Chapter 7, Exercise 2, Source: AIMA online site. (5 points)

(Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

4 Chapter 7, Exercise 9, Source: AIMA online site. (6 points)

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

1. $B \vee C$.
2. $\neg A \vee \neg B \vee \neg C \vee \neg D$.
3. $(A \implies B) \wedge A \wedge \neg B \wedge C \wedge D$.

5 Truth Tables (8 points)

Use truth tables to show that each of the following is a tautology.

1. $(p \wedge q) \rightarrow \neg(\neg p \vee \neg q)$
2. $[Mary \wedge (Mary \rightarrow Susy)] \rightarrow Susy$
3. $\alpha \rightarrow [\beta \rightarrow (\alpha \wedge \beta)]$
4. $(a \rightarrow b) \rightarrow [(b \rightarrow c) \rightarrow (a \rightarrow c)]$

6 Chapter 7, Exercise 12, Source: AIMA online site. (16 points)

Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11, Page 223).

1. $Smoke \implies Smoke$
2. $Smoke \implies Fire$
3. $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

4. $Smoke \lor Fire \lor \neg Fire$

5. $((Smoke \land Heat) \implies Fire) \Leftrightarrow ((Smoke \implies Fire) \lor (Heat \implies Fire))$

6. $Big \lor Dumb \lor (Big \implies Dumb)$

7. $(Big \land Dumb) \lor \neg Dumb$

7 Logical Equivalences (8 points)

Using a method of your choice, verify:

1. $(\alpha \rightarrow \beta) \equiv (\neg\beta \rightarrow \neg\alpha)$ contraposition

2. $\neg(\alpha \land \beta) \equiv (\neg\alpha \lor \neg\beta)$ de Morgan

3. $(\alpha \land (\beta \lor \gamma)) \equiv ((\alpha \land \gamma) \lor (\alpha \land \beta))$ distributivity of $\land$ over $\lor$

8 Chapter 7, Exercise 28, Source: AIMA online site. (18 points + 20 bonus)

Parts 1, 2, and 3 below are required. Parts 4, 5, and 6 are bonus.

Minesweeper, the well-known computer game, is closely related to the wumpus world. A minesweeper world is a rectangular grid of N squares with M invisible mines scattered among them. Any square may be probed by the agent; instant death follows if a mine is probed. Minesweeper indicates the presence of mines by revealing, in each probed square, the *number* of mines that are directly or diagonally adjacent. The goal is to probe every unmined square.

1. (Required) Let $X_{i,j}$ be true iff square $[i, j]$ contains a mine. Write down the assertion that exactly two mines are adjacent to [1,1] as a sentence involving some logical combination of $X_{i,j}$ propositions. Where [1,1] is a corner adjacent to cells: [1,2], [2,2], and [2,1].

2. (Required) Generalize your assertion from (1) by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines.

3. (Required) Explain precisely how an agent can use DPLL to prove that a given square does (or does not) contain a mine, ignoring the global constraint that there are exactly M mines in all.

4. (Bonus) Suppose that the global constraint is constructed from your method from part (2). How does the number of clauses depend on M and N? Suggest a way to modify DPLL so that the global constraint does not need to be represented explicitly.

5. (Bonus) Are any conclusions derived by the method in part (3) altered or invalidated when the global constraint is taken into account?

6. (Bonus) Give examples of configurations of probe values that induce *long-range dependencies* such that the contents of a given unprobed square would give information about the contents of a far-distant square. (*Hint*: consider an $N \times 1$ board.)

9 Proofs (28 points)

Give the explantions of each step if the steps are given, and give both the explanation and step if they are not.

- If $q \land (r \land p), t \rightarrow v, v \rightarrow \neg p$, then $\neg t \land r$.

  Proof **Explanations**

1. $q \wedge (r \wedge p)$ Given
2. $t \to v$ Given
3. $v \to \neg p$ Given
4. $t \to \neg p$
5. $(r \wedge p)$
6. r
7. p
8. $\neg\neg p$
9. $\neg t$
10. $\neg t \wedge r$

- If $p \to (q \wedge r), q \to s$, and $r \to t$, then $p \to (s \wedge t)$.

  Proof **Explanations**

  1.
  2.
  3.
  4.
  5.
  6.
  7.

- **Prove by contradiction.**

  If $\neg(\neg p \wedge q), p \to (\neg t \vee r), q$, and t, then r.

  Proof **Explanations**

  1. $\neg(\neg p \wedge q)$ Given
  2. $p \to (\neg t \vee r)$ Given
  3. q Given
  4. t Given
  5. $\neg r$ Negation of Conclusion
  6.
  7.
  8.
  9.
  10.
  11.
  12.