Title: Informed Search Methods

Required reading: AIMA, Chapter 3 (Sections 3.5 and 3.6)

LWH: Chapters 6, 10, 13 and 14.

Introduction to Artificial Intelligence

CSCE 476-876, Fall 2022

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Outline

- Categorization of search techniques
- Ordered search (search with an evaluation function)
- Best-first search:
  (1) Greedy (best-first) search (2) A*
- Admissible heuristic functions:
  how to compare them?
  how to generate them?
  how to combine them?

Types of Search (I)

1- Uninformed vs. informed

2- Systematic/constructive vs. iterative improvement

Uninformed :

use only information available in problem definition,

no idea about distance to goal

→ can be incredibly ineffective in practice

Heuristic :

exploits some knowledge of the domain

also useful for solving optimization problems

Types of Search (II)

Systematic, exhaustive, constructive search:

a partial solution is incrementally extended into global solution

Partial solution =

sequence of transitions between states

Global solution =

Solution from the initial state to the goal state

Examples:
- Uninformed
- Informed (heuristic): Greedy search, A^*

→ Returns the path; solution = path

Types of Search (III)

Iterative improvement:

A state is gradually modified and evaluated until reaching an (acceptable) optimum

→ We don't care about the path, we care about 'quality' of state

→ Returns a state; a solution = good quality state

→ Necessarily an informed search

Examples (informed):
- Hill climbing
- Simulated Annealing (physics), Taboo search
- Genetic algorithms (biology)

Ordered search

• Strategies for systematic search are generated by choosing which node from the fringe to expand first

• The node to expand is chosen by an **evaluation function**, expressing 'desirability' $\longrightarrow$ **ordered search**

• When nodes in queue are sorted according to their decreasing values by the evaluation function $\longrightarrow$ **best-first search**

• Warning: 'best' is actually 'seemingly-best' given the evaluation function. Not always best (otherwise, we could march directly to the goal!)

Search using an evaluation function

- Example: uniform-cost search!

  What is the evaluation function?

  Evaluates cost from .... to ....?

- How about the cost **<u>to</u>** the goal?

  $h(n)$ = <u>estimated</u> cost of the cheapest
  path from the state at node n to a goal state

  $h(n)$ would help focusing search

Cost to the goal

This information is not part of the problem description

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Dobreta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Best-first search

1. Greedy best-first search chooses the node n closest to the goal such as $h(n)$ is minimal

2. A* search chooses the least-cost solution

$$\text{solution cost } f(n) \begin{cases} g(n)\text{: cost from root to a given node } n \\ + \\ h(n)\text{: cost from the node } n \text{ to the goal node} \end{cases}$$

such as $f(n) = g(n) + h(n)$ is minimal

Greedy search

→ First expand the node whose state is 'closest' to the goal!

→ Minimize $h(n)$

```
function BEST-FIRST-SEARCH( problem, EVAL-FN) returns a solution sequence
   inputs: problem, a problem
           Eval-Fn, an evaluation function

   Queueing-Fn ← a function that orders nodes by EVAL-FN
   return GENERAL-SEARCH( problem, Queueing-Fn)
```

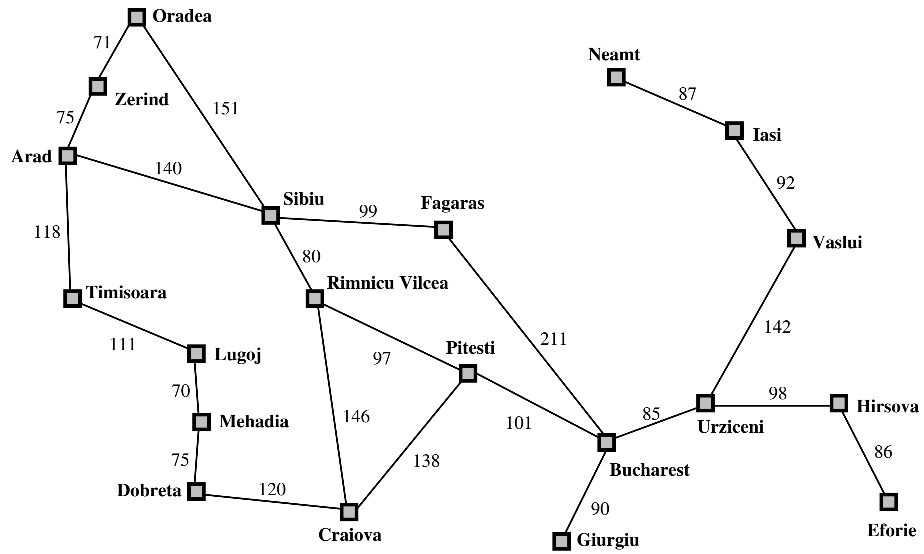
→ Usually, cost of reaching a goal may be estimated,
not determined exactly

→ If state at n is goal, $h(n)=$?

→ How to choose $h(n)$? Problem specific! Heuristic!

Greedy search: Romania

$$h_{\texttt{SLD}}(n) = \text{straight-line distance between } n \text{ and goal location}$$



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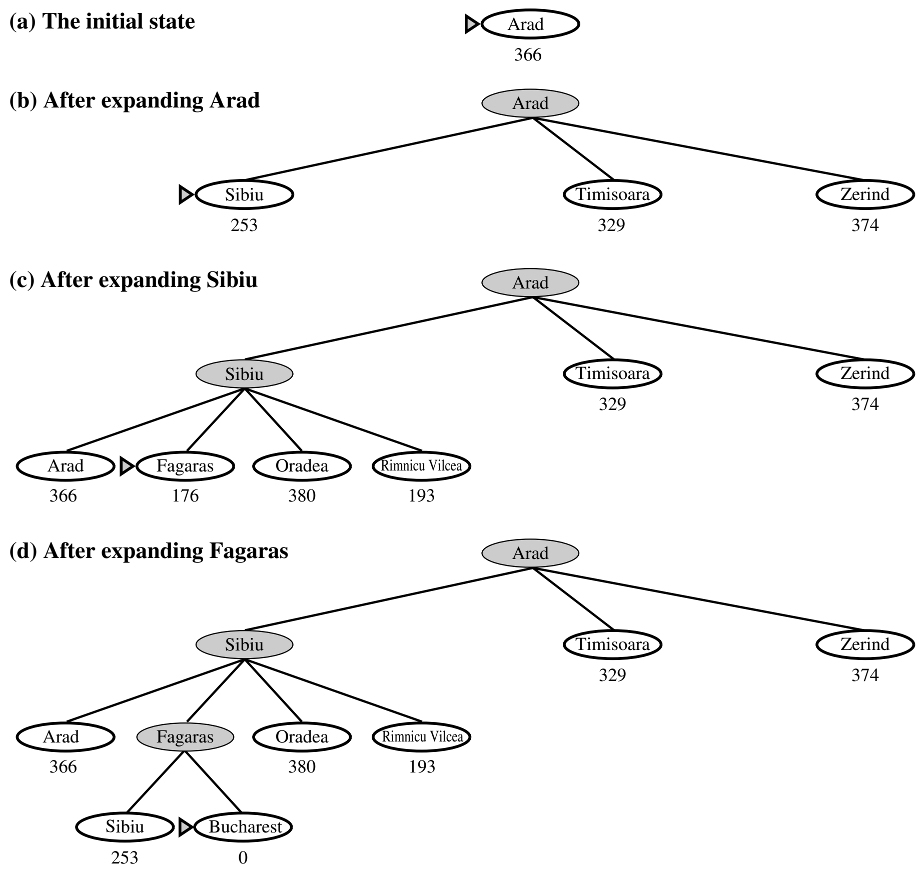
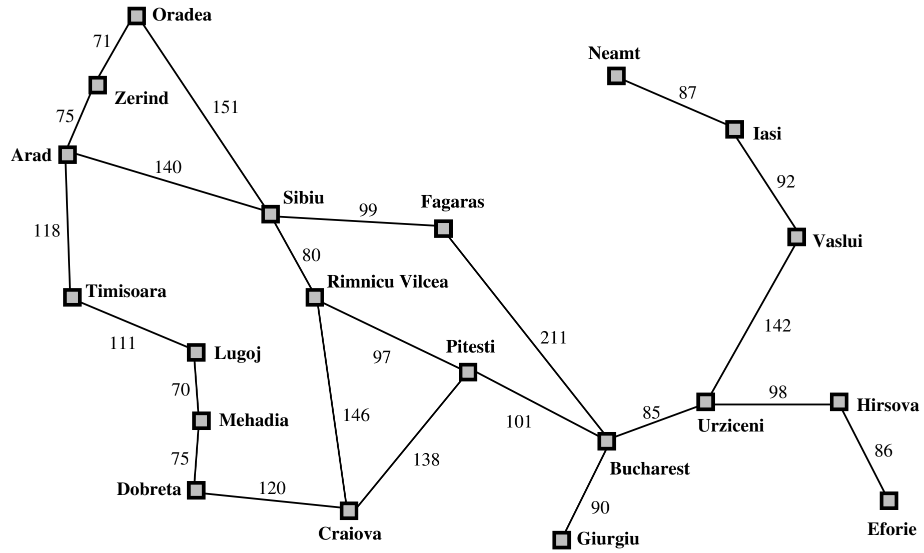
Greedy search: Trip from Arad to Bucharest



... Greedy search! quick, but not optimal!

Greedy search: Problems

From Iasi to Fagaras?
- False starts: Neamt is a dead-end
- Looping



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Greedy search: Properties

→ Like depth-first, tends to follow a single path to the goal

→ Like depth-first { Not complete; Not optimal

→ Time complexity: $O(b^m)$, m maximum depth

→ Space complexity: $O(b^m)$ retains all nodes in memory

→ Good h function (considerably) reduces space and time
but h functions are problem dependent :—(

Hmm...

Greedy search minimizes estimated cost to goal $h(n)$

→ cuts search cost considerably

→ but not optimal, not complete

Uniform-cost search minimizes cost of the path so far $g(n)$

→ is optimal and complete

→ but can be wasteful of resources

New-Best-First search minimizes $f(n) = g(n) + h(n)$

→ combines greedy and uniform-cost searches

$f(n)$ = estimated cost of cheapest solution via n

→ Provably: complete and optimal, if $h(n)$ is admissible

A* Search

- **A* search**
  Best-first search expanding the node in the fringe with minimal $f(n) = g(n) + h(n)$
- **A* search with admissible** $h(n)$
  Provably complete, optimal using TREE-SEARCH
- **A* search with consistent** $h(n)$
  Provably optimally efficient using TREE-SEARCH
  Remains optimal even using GRAPH-SEARCH

(See TREE-SEARCH versus GRAPH-SEARCH page 77)

Admissible heuristic

An admissible heuristic is a heuristic that never overestimates the cost to reach the goal from the current node: $h(n) \leq h^*(n)$

$\rightarrow$ is optimistic

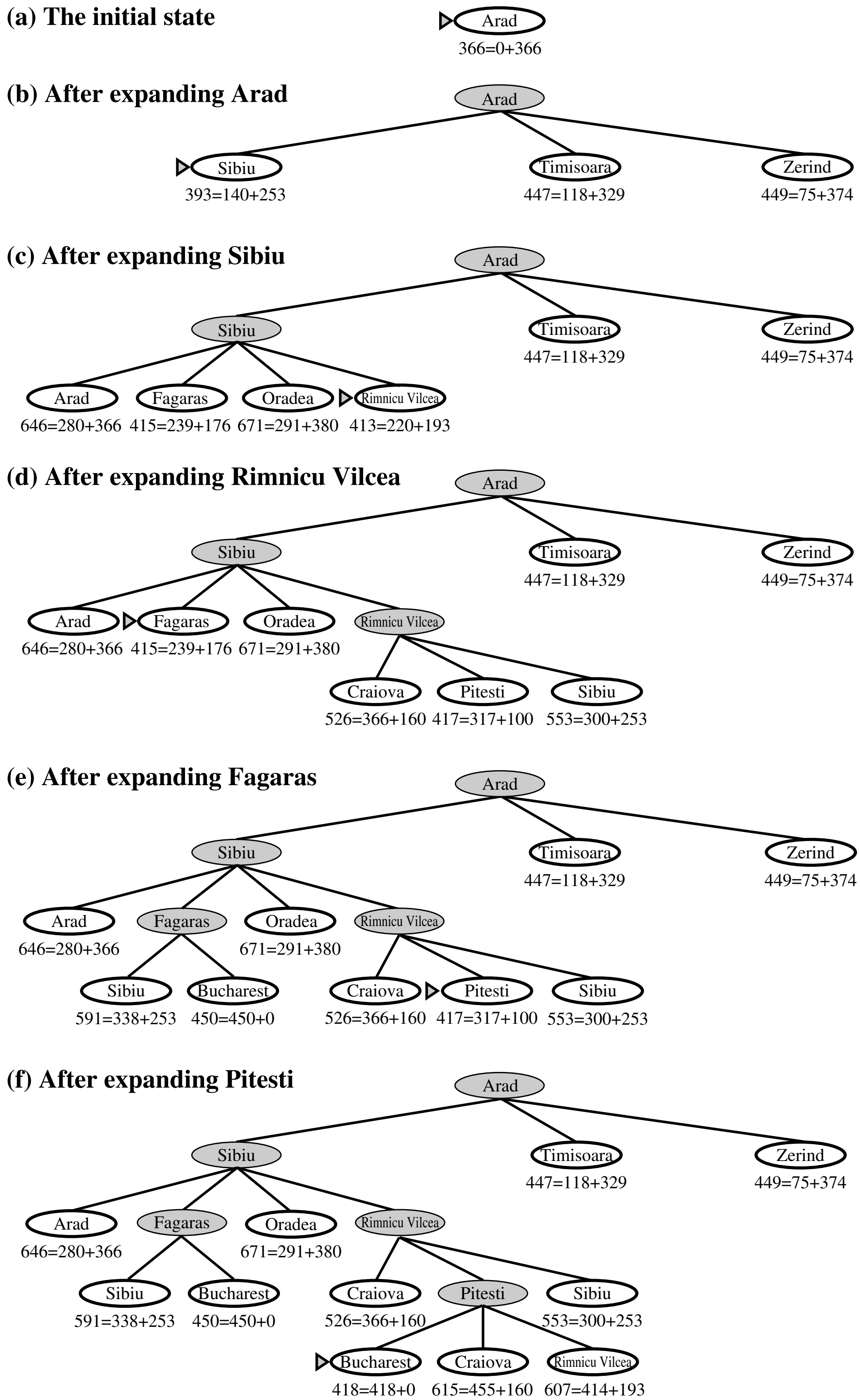
$\rightarrow$ thinks the cost of solving is less than it actually is

Example:
- travel: straight line distance
- I need 3 years to finish college (at least!)
- We are 3 years away from the first flight to Mars (at least!)

If h is admissible,

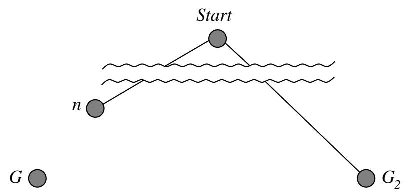
$f(n)$ never overestimates the actual cost of the best solution through n ($f(n) \leq f^*(n)$)

A* Search From Arad to Bucharest



A* Search is optimal

- G, G_2 goal states $\Rightarrow h(G) = h(G_2) = 0 \Rightarrow g(G) = f(G)$ and $f(G_2) = g(G_2)$
- G optimal goal state $\Rightarrow C^* = f(G)$
- G_2 suboptimal $\Rightarrow f(G_2) > C^* = f(G)$ (1)
- Suppose n is not chosen for expansion



- $f(n) > C^*$ otherwise n would have been expanded
- $f(n) = g(n) + h(n)$ by definition
- $f(n) = g^*(n) + h(n)$ because n is on an optimal path
- We know that $f(n) \leq g^*(n) + h^*(n)$ because $h(n) \leq h^*(n)$, h is admissible
- Thus, $f(n) \leq C^*$ because $C^* = g^*(n) + h^*(n)$

We get a contradiction, thus, n should be chosen for expansion

Which nodes does A* expand?

GOAL-TEST is applied to STATE(node) when a node is chosen from the fringe for expansion, not when the node is generated

Theorem 3 & 4 in Pearl 84, original results by Nilsson

- *Necessary condition:* Any node expanded by A* cannot have an f value exceeding C^*: For all nodes expanded, $f(n) \leq C^*$
- *Sufficient condition:* Every node in the fringe with $f(n) < C^*$ will eventually be expanded by A*

In summary

- A* expands no nodes with $f(n) > C^*$
- A* expands some nodes with $f(n) = C^*$
- All nodes expanded by A* are $f(n) \leq C^*$

A* **Search** is complete

Completeness is guaranteed as long as A* expands only a finite number of nodes n with $f(n) \leq C^*$, unless

$$\begin{cases} \text{1. } \exists \text{ a node with infinite branching factor} \\ \text{or} \\ \text{2. } \exists \text{ a path with infinite number of nodes along it} \end{cases}$$

$$\text{A}^* \text{ is complete} \begin{cases} \text{on locally finite graphs} \\ \text{and} \\ \exists \delta > 0 \text{ constant, the cost of each operator} > \delta \end{cases}$$

A* **Search** Complexity

Time:

Exponential in (relative error in h $\times$ length of solution path)
... quite bad

Space: must keep all nodes in memory

Number of nodes within goal contour is exponential in length of solution.... unless the error in the heuristic function $|h(n) - h^*(n)|$ grows no faster than the log of the actual path cost: $|h(n) - h^*(n)| \leq O(\log h^*(n))$

In practice, the error is proportional... impractical..

major drawback of A*: runs out of space quickly

→ Memory Bounded Search IDA*(not addressed here)

Tree-Search vs. Graph-Search

After choosing a node from the fringe and before expanding it, Graph-Search checks whether State(node) was visited before to avoid loops.

→ Graph-search may lose optimal solution

Solutions

1. In Graph-Search, discard the more expensive path to a node
2. Ensure that the optimal path to any repeated state is the first one found
   → Consistency

Consistency

$h(n)$ is consistent

If $\forall\ n$ and $\forall\ n'$ successor of n generated by action a, we have
$h(n) \leq c(n, a, n') + h(n')$, n' is an immediate successor of n
Triangle inequality ($\langle n, n'$, goal$\rangle$)

Monotonicity

$h(n)$ is monotone

If $\forall\ n$ and $\forall\ n'$ successor of n along a path, we have
$h(n) \leq k(n, n') + h(n')$, k cost of cheapest path from n to n'

Important: h is consistent $\Leftrightarrow$ h is monotone

Beware: of confusing terminology 'consistent' and 'monotone'
Values of h not necessarily decreasing/nonincreasing

A^* with a consistent heuristic is optimally efficient

.. for any given evaluation function: no other algorithms that finds the optimal solution is guaranteed to expend fewer nodes than A^*

Interpretation (proof not presented): Any algorithm that does not expand all nodes between root and the goal contour risks missing the optimal solution

History: Initially, an admissible heuristic was thought to guarantee an optimally efficient search, Dechter and Pearl (JACM) 1985) showed that consistency is needed.

Properties of h: Important results

- h consistent $\Leftrightarrow$ h monotone (Pearl 84)
- h consistent $\Rightarrow$ h admissible (AIMA, Exercise 4.7)
  consistency is stricter than admissibility
- h consistent $\Rightarrow$ f is nondecreasing
  - By definition: $f(n') = g(n') + h(n')$
  - By definition: $g(n') = g(n) + c(n, a, n')$
  - Thus, $f(n') = g(n) + c(n, a, n') + h(n')$
  - Because f consistent: $c(n, a, n') + h(n') \geq h(n)$
  - Thus, $f(n') \geq g(n) + h(n) = f(n)$
- h consistent $\Rightarrow$ A* using TREE-SEARCH is optimally efficient
- h consistent $\Rightarrow$ A* using GRAPH-SEARCH is optimal

Nondecreasing evaluation function

The evaluation function f is guaranteed nondecreasing if and only if h is consistent/monotone

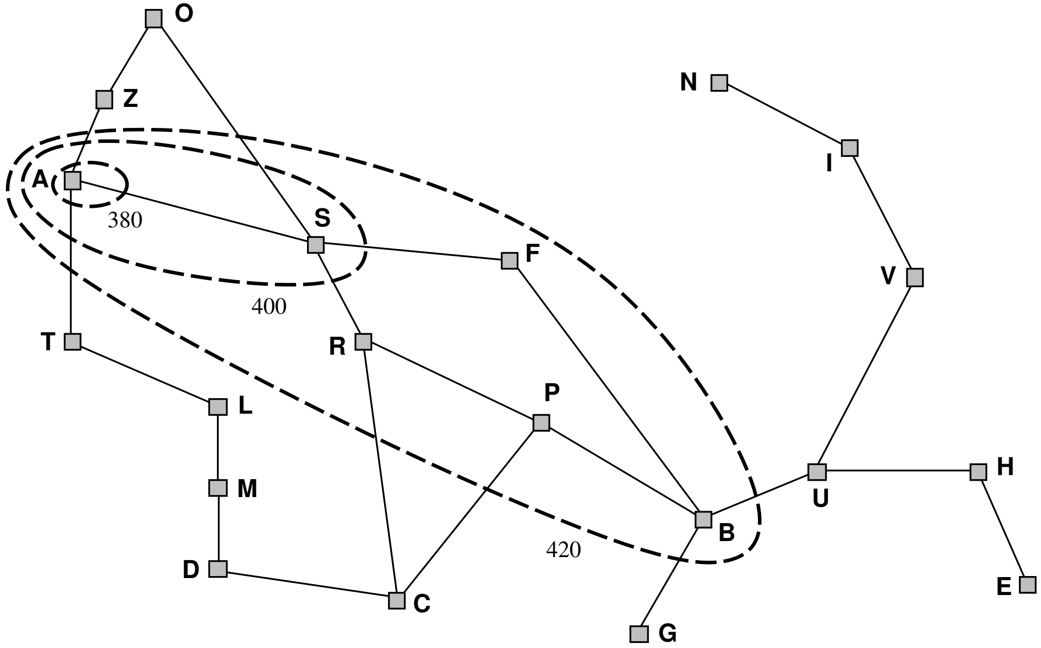
When f is nondecreasing, we have

- A^* expands no nodes with $f(n) > C^*$
- A^* expands some nodes with $f(n) = C^*$
- A^* expands all nodes with $f(n) < C^*$
  (contrast to previous statement: All nodes expanded by A^* are $f(n) \leq C^*$)

Expanding contours

When f is non-decreasing, A* expands nodes from fringe in increasing f value

We can conceptually draw contours in the search space



The first solution found is necessarily the optimal solution

Careful: a Test-Goal is applied at node expansion

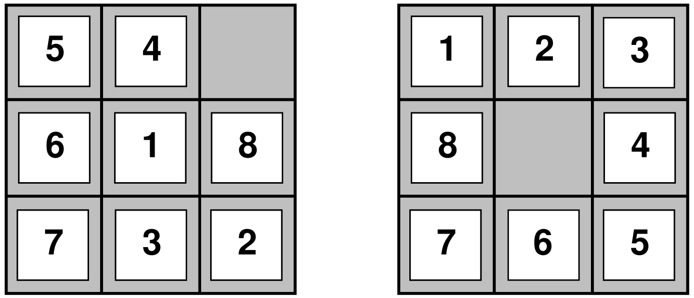
Summarizing definitions for A*

- A* is a best-first search that expands the node in the fringe with minimal $f(n) = g(n) + h(n)$
- An admissible function h never overestimates the distance to the goal.
- h admissible $\Rightarrow$ A* is complete and optimal using TREE-SEARCH
- h consistent $\Leftrightarrow$ h monotone
  h consistent $\Rightarrow$ h admissible
  h consistent $\Rightarrow$ f nondecreasing
- h consistent $\Rightarrow$ A* is optimally efficient TREE-SEARCH
- h consistent $\Rightarrow$ A* remains optimal using GRAPH-SEARCH

Admissible heuristic functions

Examples

- Route-finding problems: straight-line distance
- 8-puzzle: $\begin{cases} h_1(n) = \text{number of misplaced tiles} \\ h_2(n) = \text{total Manhattan distance} \end{cases}$



Start State Goal State

$h_1(S) = ?$

$h_2(S) = ?$

Performance of admissible heuristic functions

Two criteria to compare admissible heuristic functions:

1. Effective branching factor: b^*
2. Dominance: number of nodes expanded

Effective branching factor b^*

– The heuristic expands N nodes in total

– The solution depth is d

$\longrightarrow$ b^* is the branching factor had the tree been uniform

$$N = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d = \frac{(b^*)^{d+1} - 1}{b^* - 1}$$

– Example: N=52, d=5 $\rightarrow$ $b^* = 1.92$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs: nodes expanded

Sol. depth	**IDS**	$\mathbf{A}^*(h_1)$	$\mathbf{A}^*(h_2)$
$d = 12$	3,644,035	227	73
$d = 24$	too many	39,135	1,641

A* expands all nodes $f(n) < C^* \Rightarrow g(n) + h(n) < C^*$
$\Rightarrow h(n) < C^* - g(n)$

If $h_1 \leq h_2$, A* with h_1 will always expand at least as many (if not more) nodes than A* with h_2

$\longrightarrow$ It is always better to use a heuristic function with higher values, as long as it does not overestimate (remains admissible)

How to generate admissible heuristics?

→ Use *exact* solution cost of a relaxed (easier) problem

Steps:

– Consider problem P

– Take a problem P' easier than P

– Find solution to P'

– Use solution of P' as a heuristic for P

Relaxing the 8-puzzle problem

A tile can move mode square A to square B if

A is (horizontally or vertically) adjacent to B and B is blank

1. A tile can move from square A to square B if A is adjacent to B
   The rules are relaxed so that a tile can move to *any adjacent square*: the shortest solution can be used as a heuristic ($\equiv h_2(n)$)
2. A tile can move from square A to square B if B is blank
   Gaschnig heuristic (Exercice 3.31, AIMA, page 119)
3. A tile can move from square A to square B
   The rules of the 8-puzzle are relaxed so that a tile can move *anywhere*: the shortest solution can be used as a heuristic ($\equiv h_1(n)$)

An admissible heuristic for the TSP

Let path be *any* structure that connects all cities
$\Longrightarrow$ minimum spanning tree heuristic (polynomial)

(Exercice 3.30, AIMA, page 119)

Combining several admissible heuristic functions

We have a set of admissible heuristics $h_1, h_2, h_3, \ldots, h_m$ but no heuristic that dominates all others, what to do?

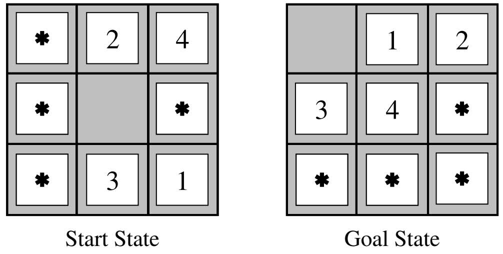
$$\longrightarrow h(n) = \texttt{max}(h_1(n), h_2(n), \ldots, h_m(n))$$

h is admissible and dominates all others.

$\rightarrow$ Problem:
Cost of computing the heuristic (vs. cost of expanding nodes)

Using subproblems to derive an admissible heuristic function

Goal: get 1, 2, 3, 4 into their correct positions, ignoring the 'identity' of the other tiles



Cost of optimal solution to subproblem used as a lower bound (and is substantially more accurate than Manhattan distance)

Pattern databases:

- Identify patterns (which represent several possible states)
- Store cost of <u>exact</u> solutions of patterns
- During search, retrieve cost of pattern and use as a (tight) estimate

Cost of building the database is amortized over 'time'

Other techniques

- Disjoint pattern databases: combining heuristics of two patterns provided adimissibility is preserved
- Precomputation of some optimal paths (e.g., maps), cost amortized over time
  Example 1: precomputing optimal path betweeen every two pairs of cities
  Example 2: Choose some landmark cities; for each city v and each landmark L, compute and store $C^*(v, L)$

  $$h_L(n) = min_{L \in \text{Landmarks}} C^*(n, L) + C^*(L, \text{goal})$$

  If optimal path goes through L, h_L is exact, otherwise it is not admissible.
- More techniques in textbook..