

Title: First-Order Logic

AIMA: Chapter 8 (Sections 8.1 and 8.2)

Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence

CSCE 476-876, Fall 2022

URL: www.cse.unl.edu/~choueiry/F22-476-876

Berthe Y. Choueiry (Shu-we-ri)

(402)472-5444

Outline

- First-order logic:
 - basic elements
 - syntax
 - semantics
- Examples

Pros and cons of propositional logic

- Propositional logic is declarative: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
(unlike most data structures and databases)
- Propositional logic is compositional:
meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent
(unlike natural language, where meaning depends on context)
- but...
Propositional logic has very limited expressive power
E.g., cannot say “pits cause breezes in adjacent squares”
except by writing one sentence for each square

Propositional Logic

- is simple
- illustrates important points:
model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness:
→ In PL, world contains facts

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)

First Order Logic

- FOL provides more "primitives" to express knowledge:
- objects (identity & properties)
 - relations among objects (including functions)

5

Objects: people, houses, numbers, Einstein, Huskers, event, ..

Properties: smart, nice, large, intelligent, loved, occurred, ..

Relations: brother-of, bigger-than, part-of, occurred-after, ..

Functions: father-of, best-friend, double-of, ..

Examples: (objects? function? relation? property?)

— one plus two equals four [sic]

— squares neighboring the wumpus are smelly

Logic

Attracts: mathematicians, philosophers and AI people

Advantages:

- allows to represent the world and reason about it
- expresses anything that can be programmed

Non-committal to:

- symbols could be objects or relations
(*e.g.*, King(Gustave), King(Sweden, Gustave), Merciless(King))
- classes, categories, time, events, uncertainty

.. but amenable to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

→ Some people think FOL **is** the language of AI
true/false? donno :—(but it will remain around for some time..

Types of logic

Logics are characterized by what they commit to as “primitives”

Ontological commitment :

what exists—facts? objects? time? beliefs?

Epistemological commitment :

what states of knowledge?

7

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0...1
Fuzzy logic	degree of truth	degree of belief 0...1

Higher-Order Logic: views relations and functions of FOL as objects

Syntax of FOL: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: x , y , etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation) Father-of, Square-root, LeftLeg, etc.
- Quantifiers \forall , \exists
- Connectives: \wedge , \vee , \neg , \Rightarrow , \Leftrightarrow ,
- (Sometimes) equality =

Predicates and functions can have any arity (number of arguments)

Basic elements in FOL (i.e., the grammar)

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
 - atomic sentences
 - complex sentences
- Quantifiers:
 - Universal quantifier
 - Existential quantifier

Term

logical expression that refers to an object

- built with: constant symbols, variables, function symbols

$$\text{Term} = \textit{function}(\textit{term}_1, \dots, \textit{term}_n)$$

or constant or variable

- **ground term**: term with no variable

Atomic sentences

state facts

built with terms and predicate symbols

Atomic sentence = $predicate(term_1, \dots, term_n)$
or $term_1 = term_2$

Examples:

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))

Complex Sentences

built with atomic sentences and logical connectives

$$\neg S$$

$$S_1 \wedge S_2$$

$$S_1 \vee S_2$$

$$S_1 \Rightarrow S_2$$

$$S_1 \Leftrightarrow S_2$$

Examples:

$\text{Sibling}(\text{KingJohn}, \text{Richard}) \Rightarrow \text{Sibling}(\text{Richard}, \text{KingJohn})$

$$\gt(1, 2) \vee \leq(1, 2)$$

$$\gt(1, 2) \wedge \neg \gt(1, 2)$$

Truth in first-order logic: Semantic

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for

constant symbols \rightarrow objects

predicate symbols \rightarrow relations

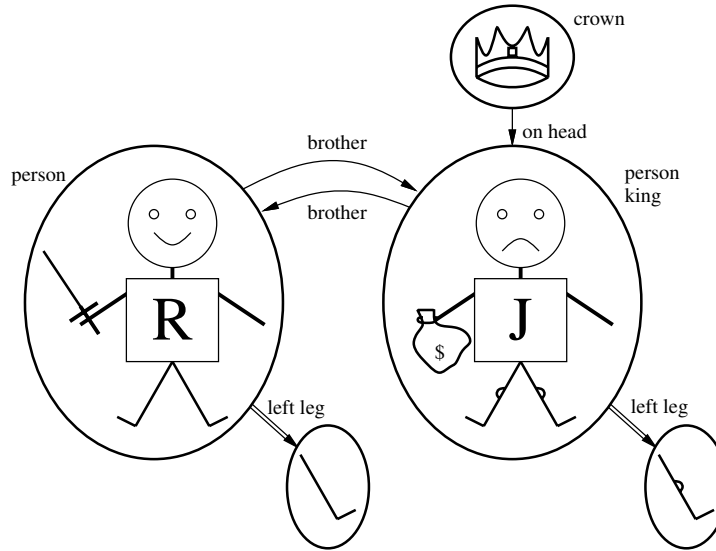
function symbols \rightarrow functional relations

An atomic sentence $predicate(term_1, \dots, term_n)$ is true

iff the objects referred to by $term_1, \dots, term_n$

are in the relation referred to by *predicate*

Model in FOL: example



The domain of a model is the set of objects it contains:
five objects

Intended interpretation: Richard refers Richard the Lion Heart,
John refers to Evil King John, Brother refers to brotherhood
relation, etc.

Models for FOL: Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞

For each k -ary predicate P_k in the vocabulary

For each possible k -ary relation on n objects

For each constant symbol C in the vocabulary

For each choice of referent for C from n objects ...

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded

→ Checking entailment by enumerating is not an option

Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things

Universal quantification

\forall *<variables>* *<sentence>*

Example: all dogs like bones $\forall x \text{Dog}(x) \Rightarrow \text{LikeBones}(x)$

$x = \text{Indy}$ is a dog

$x = \text{Indiana Jones}$ is a person

$\forall x P$ is equivalent to the conjunction of instantiations of P

$\text{Dog}(\text{Indy}) \Rightarrow \text{LikeBones}(\text{Indy})$

$\wedge \text{Dog}(\text{Rebel}) \Rightarrow \text{LikeBones}(\text{Rebel})$

$\wedge \text{Dog}(\text{KingJohn}) \Rightarrow \text{LikeBones}(\text{KingJohn})$

$\wedge \dots$

Typically: \Rightarrow is the main connective with \forall

Common mistake: using \wedge as the main connective with \forall

Example: $\forall x \text{Dog}(x) \wedge \text{LikeBones}(x)$

all objects in the world are dogs, and all like bones

Existential quantification

$\exists \langle \text{variables} \rangle \langle \text{sentence} \rangle$

Example: some student will talk at the TechFair

$\exists x \text{Student}(x) \wedge \text{TalksAtTechFair}(x)$

Pat, Leslie, Chris are students

$\exists x P$ is equivalent to the disjunction of instantiations of P

$\text{Student}(\text{Pat}) \wedge \text{TalksAtTechFair}(\text{Pat})$

$\vee \text{Student}(\text{Leslie}) \wedge \text{TalksAtTechFair}(\text{Leslie})$

$\vee \text{Student}(\text{Chris}) \wedge \text{TalksAtTechFair}(\text{Chris})$

$\vee \dots$

Typically: \wedge is the main connective with \exists

Common mistake: using \Rightarrow as the main connective with \exists

$\exists x \text{Student}(x) \Rightarrow \text{TalksAtTechFair}(x)$

is true if there is anyone who is not Student

Properties of quantifiers (I)

$\forall x \forall y$ is the same as $\forall y \forall x$

$\exists x \exists y$ is the same as $\exists y \exists x$

$\exists x \forall y$ is not the same as $\forall y \exists x$

$\exists x \forall y \text{ Loves}(x, y)$

“There is a person who loves everyone in the world”

$\forall y \exists x \text{ Loves}(x, y)$

“Everyone in the world is loved by at least one person”

Quantifier duality: each can be expressed using the other

$\forall x \text{ Likes}(x, \text{IceCream}) \quad \neg \exists x \neg \text{Likes}(x, \text{IceCream})$

$\exists x \text{ Likes}(x, \text{Broccoli}) \quad \neg \forall x \neg \text{Likes}(x, \text{Broccoli})$

Parsimony principal: \forall , \neg , and \Rightarrow are sufficient

Properties of quantifiers (II)

Nested quantifier:

$\forall x(\exists y(P(x, y)):$

every object in the world has a particular property, which is the property to be related to some object by the relation P

$\exists x(\forall y(P(x, y)):$

there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

Lexical scoping: $\forall x[Cat(x) \vee \exists x Brother(Richard, x)]$

Well-formed formulas (WFF): (kind of correct spelling)

every variable must be introduced by a quantifier

$\forall xP(y)$ is not a WFF

Examples

Brothers are siblings

.

“Sibling” is symmetric

.

One’s mother is one’s female parent

.

A first cousin is a child of a parent’s sibling

Examples

.

$$\forall x, y \text{ Brother}(x, y) \Rightarrow \text{Sibling}(x, y)$$

.

$$\forall x, y \text{ Sibling}(x, y) \Rightarrow \text{Sibling}(y, x)$$

.

$$\forall x, y \text{ Mother}(x, y) \Leftrightarrow (\text{Female}(x) \wedge \text{Parent}(x, y))$$

.

$$\forall x, y \text{ FirstCousin}(x, y) \Leftrightarrow$$

$$\exists a, b \text{ Parent}(a, x) \wedge \text{Sibling}(a, b) \wedge \text{Parent}(b, y)$$

Tricky example

Someone is loved by everyone

$$\exists x \forall y \text{ Loves}(y, x)$$

Someone with red-hair is loved by everyone

$$\exists x \forall y \text{ Redhair}(x) \wedge \text{ Loves}(y, x)$$

Alternatively:

$$\exists x \text{ Person}(x) \wedge \text{ Redhair}(x) \wedge (\forall y \text{ Person}(y) \Rightarrow \text{ Loves}(y, x))$$

Equality

$term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

Example: $Father(John) = Henry$

Equality is useful to distinguish two objects. Examples:

- Definition of (full) *Sibling* in terms of *Parent*.

$$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow [\exists m, f \neg(m = f) \wedge \text{Parent}(m, x) \wedge \text{Parent}(f, x) \wedge \text{Parent}(m, y) \wedge \text{Parent}(f, y)]$$

- Spot has at least two sisters: ...

AIMA, Exercise 8.4. Write: “All Germans speak the same languages,” where $Speaks(x, l)$ means that person x speaks language l .

Knowledge representation (KR)

Domain: a section of the world about which we wish to express some knowledge

Example: Family relations (kinship):

- Objects: people
- Properties: gender, married, divorced, single, widowed
- Relations: parenthood, brotherhood, marriage..

Unary predicates: Male, Female

Binary relations: Parent, Sibling, Brother, Child, etc.

Functions: Mother, Father

$$\forall m, c, \text{Mother}(c) = m \Leftrightarrow \text{Female}(m) \wedge \text{Parent}(m, c)$$

In Logic (informally)

- Basic facts: axioms (definitions)
- Derived facts: theorems

Independent axiom

an axiom that cannot be derived from the rest

→ Goal of mathematicians: find the minimal set
of independent axioms

In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$

$Ask(KB, \exists a Action(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: *Yes*, $\{a/Shoot\}$ \leftarrow substitution (binding list)

Given a sentence S and a substitution σ ,

$S\sigma$ denotes the result of plugging σ into S ; e.g.,

$S = Smarter(x, y)$

$\sigma = \{x/Hillary, y/Bill\}$

$S\sigma = Smarter(Hillary, Bill)$

$Ask(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Prepare for next lecture: AIMA, Exercise 8.24, page 319

$\text{Takes}(x, c, s)$: student x takes course c in semester s

$\text{Passes}(x, c, s)$: student x passes course c in semester s

$\text{Score}(x, c, s)$: the score obtained by student x in course c in semester s

$x > y$: x is greater than y

F and G : specific French and Greek courses

$\text{Buys}(x, y, z)$: x buys y from z

$\text{Sells}(x, y, z)$: x sells y from z

$\text{Shaves}(x, y)$: person x shaves person y

$\text{Born}(x, c)$: person x is born in country c

$\text{Parent}(x, y)$: person x is parent of person y

$\text{Citizen}(x, c, r)$: person x is citizen of country c for reason r

$\text{Resident}(x, c)$: person x is resident of country c of person y

$\text{Fools}(x, y, t)$: person x fools person y at time t

$\text{Student}(x)$, $\text{Person}(x)$, $\text{Man}(x)$, $\text{Barber}(x)$, $\text{Expensive}(x)$, $\text{Agent}(x)$,

$\text{Insured}(x)$, $\text{Smart}(x)$, $\text{Politician}(x)$,

AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986

head of AI at AT&T Labs-Research

Program co-chair of AAAI-2000

Professor at University of Washington, Seattle

Founding Director of Institute for Data Science and Professor at University of Rochester