Homework 8

Assigned on: Monday, November 21, 2022.

Due: Wednesday, December 7, 2022.

This homework is a pen-and-paper homework, to be returned in class or with web handin. The homework is worth 146 points, plus 40 bonus points.

Exercises: AIMA exercises are available online : "https://aimacode.github.io/aima-exercises"

Contents

1 Using the inference rules for logic (10 points)

prove that "$\exists\, x\, Z(x)$ follows from the givens." Be sure to justify your steps by stating the inference rule used, along with the previous line(s) to which it was applied and the unifications used.

1.	$P(1)$	given
2.	$W(1) \wedge W(2) \wedge W(3)$	given
3.	$\forall\, x\, [P(x) \Rightarrow \neg R(x)]$	given
4.	$\forall\, x\, [Q(x) \vee R(x)]$	given
5.	$\forall\, x\, [(Q(x) \wedge W(x)) \Rightarrow Z(x)]$	given

2 Chapter 9, Exercise 3, Source: AIMA online site. (3 points)

Suppose a knowledge base contains just one sentence, $\exists\, x\, AsHighAs(x, Everest)$. Which of the following are legitimate results of applying Existential Instantiation?

1. $AsHighAs(Everest, Everest)$.

2. $AsHighAs(Kilimanjaro, Everest)$.

3. $AsHighAs(Kilimanjaro, Everest) \wedge AsHighAs(BenNevis, Everest)$
   (after two applications).

3 Chapter 9, Exercise 4, Source: AIMA online site. (4 points)

For each pair of atomic sentences, give the most general unifier if it exists:

1. $P(A, B, B)$, $P(x, y, z)$.

2. $Q(y, G(A, B))$, $Q(G(x, x), y)$.

3. $Older(Father(y), y)$, $Older(Father(x), John)$.

4. $Knows(Father(y), y)$, $Knows(x, x)$.

4 Chapter 9, Exercise 7, Source: AIMA online site. (12 points)

Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

1. Horses, cows, and pigs are mammals.

2. An offspring of a horse is a horse.

3. Bluebeard is a horse.

4. Bluebeard is Charlie's parent.

5. Offspring and parent are inverse relations.

6. Every mammal has a parent.

5 Chapter 9, Exercise 16, Source: AIMA online site. (12 points)

In this exercise, use the sentences you wrote in Chapter 9, Exercise 7 (Previous Question) to answer a question by using a backward-chaining algorithm.

1. Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $\exists h \, Horse(h)$, where clauses are matched in the order given.

2. What do you notice about this domain?

3. How many solutions for h actually follow from your sentences?

4. Can you think of a way to find all of them? (*Hint*: For this question, it is usefull to check the following paper https://cse.unl.edu/ choueiry/Documents/Smith-Genesereth-AIJ86.pdf Smith, D.E., Genesereth, M.R., and Ginsberg, M.L. (1986). *Controlling recursive inference.* Artificial Intelligence, Volume 30(3), pages 343–389.)

6 First-Order Logic   (20 points)

Consider the following axioms:

1. Anyone who rides any Harley is a rough character.

2. Every biker rides [something that is] either a Harley or a BMW.

3. Anyone who rides any BMW is a yuppie.

4. Every yuppie is a lawyer.

5. Any nice girl does not date anyone who is a rough character.

6. Mary is a nice girl, and John is a biker.

7. (Conclusion) If John is not a lawyer, then Mary does not date John.

- Choose appropriate predicates to write the above axioms in first-order logic, clearly indicating the arguments and arity of each predicate: (2 points)

- Write each of the above axioms in first-order logic. Use scratch paper if necessary, and *neatly* report your results below. (10 points)

   1.

   2.

   3.

   4.

   5.

   6.

   7.

- Transform each of the above sentences into a conjunctive normal form. Clearly state the Skolem functions and clearly number the statements. (4 points)

- Establish the conclusion using the axioms by applying refutation resolution. Clearly show the variable bindings at each step and clearly number the statements.

   (4 points)

   Negation of conclusion:

7   Unification (4 points)

What is the most general unifier of the following pairs of wff's? If none exists, report "fail." Assume that the capital letters are constants and the lowercase letters are variables.

1. $P(x, y, x, z)$ and $P(F(w), A, F(B), w)$

2. $Q(x, F(x), G(F(x)))$ and $Q(1, y, G(F(y)))$

3. $\text{Foo}(x, y)$ and $\text{Foo}(y, x)$

4. $\text{Mother}(x, y)$ and $\text{Mother}(y, \text{Father}(x))$

8 Unification and Resolution (2 points)

You are given the following pairs of clauses where upper case letters indicate constants and lower case letters indicate variables, functions, or predicates. Consider each pair independently of the others. In each pair, variables with the same name are meant to be the same variable. For each of the pairs, specify if the two clauses can be <u>resolved</u>. If yes, show the results of the unification process. If not, explain why.

1. $p(B, C, x, z, f(A, z, B))$ and $\neg p(y, z, y, C, w)$.

2. $r(f(y), y, x)$ and $\neg r(x, f(A), f(v))$.

9 Resolution and Refutation (5 points)

Use resolution and refutation to solve the problem below. Hint: First transform the givens into clausal form.
Given:

1. $\forall x(P(x) \Rightarrow Q(x))$

2. $\forall x(P(x) \Rightarrow (\forall y W(y)))$

3. $\forall x \forall y((Q(x) \wedge W(y)) \Rightarrow S(x))$

4. $P(Mary)$

Show: $S(Mary)$

10 Translation into FOL (14 points)

Consider the set of all creatures. We will use the following predicates:

- $Insect(x)$.

- $Moth(x)$.

- $Dragonfly(x)$.

- $Spider(x)$.

- $Eats(x, y)$.

- $Wings(x, y)$: x has y wings.

- $Order(x)$. (Recall that zoologists classify creatures using kingdom, phylum, class, order, family, genus, and species.)

Translate the following sentences into logic. You may only use the identity predicate and the predicates and functions listed above. Try not to include more bugs in your logic than are required..

- Flik is an insect with 4 wings but is not a moth.

- Not all insects have 4 wings.

- All insects with 2 wings are in the same order.

- There are at least 3 difference orders.

- Moths and Dragonflies are insects but are not in the same order.

- All spiders eat insects.

- Some spiders eat only insects.

11 Inference in First-Order Logic: CNF and Resolution (60 points + 40 Bonus)

Choose *three* (3) of the exercices below, and for each of them, answer each of the following questions. The remaining two exercises are bonus, each worth 20 points.

- Choose appropriate predicates to write the above axioms in first-order logic.

- Write the axioms in First-Order Logic. Report your results neatly.

- Transform each of the first-order sentences into Conjunctive Normal Form. Clearly state the Skolem functions and clearly number the statements. Neatly report your results and provide as much detail as possible.

- Establish the conclusion using the axioms by applying *refutation resolution*. That is, negate the conclusion and prove the unsatisfiability of the set of clauses by resolution. Clearly show the variable bindings at each step and clearly number the statements.

11.1 Do Loons Eat Fish? (20 points)

Consider the following axioms.

1. Every bird sleeps in some tree.

2. Every loon is a bird, and every loon is aquatic.

3. Every tree in which any aquatic bird sleeps is beside some lake.

4. Anything that sleeps in anything that is beside any lake eats fish.

5. (Conclusion) Every loon eats fish.

11.2   Is there a conservative Austinite? (20 points)

Consider the following axioms:

1. Every Austinite who is not conservative loves some armadillo.

2. Anyone who wears maroon-and-white shirts is an Aggie.

3. Every Aggie loves every dog.

4. Nobody who loves every dog loves any armadillo.

5. Clem is an Austinite, and Clem wears maroon-and-white shirts.

6. (Conclusion) Is there a conservative Austinite?

11.3   Will Mary Date John? (20 points)

Consider the following axioms:

1. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.

2. Every dog chases some rabbit.

3. Mary buys carrots by the bushel.

4. Anyone who owns a rabbit hates anything that chases any rabbit.

5. John owns a dog.

6. Someone who hates something owned by another person will not date that person.

7. (Conclusion) If Mary does not own a grocery store, she will not date John.

11.4   Drunk AI Students (20 points)

Consider the following axioms.

1. Everyone who feels warm either is drunk, or every costume they have is warm.

2. Every costume that is warm is furry.

3. Every AI student is a CS student.

4. Every AI student has some robot costume.

5. No robot costume is furry.

6. (Conclusion) If every CS student feels warm, then every AI student is drunk.

11.5 Brilliant CS Students (20 points)

Consider the following axioms:

1. Every student who makes good grades is brilliant or studies.

2. Every student who is a CS major has some roommate. [Make "roommate" a two-place predicate.]

3. Every student who has any roommate who likes to party goes to Sixth Street.

4. Anyone who goes to Sixth Street does not study.

5. (Conclusion) If every roommate of every CS major likes to party, then every student who is a CS major and makes good grades is brilliant.