Homework 7

Assigned on: Friday, Nov 5, 2021

Due: ~~Wednesday, Nov 24, 2021~~ Monday, Nov 29, 2021

This is a pen-and-paper homework, to be returned in class or with web handin.
The homework is worth 194 points for ugrads and 214 points for grads (+85 bonus points for ugrads and +65 bonus points for grads).

Exercises: AIMA exercises are available online : https://aimacode.github.io/aima-exercises

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1 Bonus Researching Description Logic (Bonus 25 points)

Description Logic is a cornerstone of the Semantic Web technology. In this question, you are asked to research Description Logic *beyond what is in your textbook*. Write a two-page (typed) structured summary about DL addressing whatever aspects you find meaningful and interesting. Below is a list of ideas *you may want to include*, they are mere suggestions. Make sure you cite all your references.

1. What is the goal of DL?
2. To the extent possible, explain/state the syntax and semantics of DL.
3. How does DL relate to other types of Logic that we may or may not have studies?
4. Explain some proof techniques used for DL and give their complexity.
5. Briefly describe the history/evolution of DL.
6. Discuss and compare various implementations of DL.
7. Investigate the industrial impact of DL: list practical systems implements some version of DL; are they public domain; have they generated economic growth/benefit, etc.

2 Algorithms for Propositional Logic (Mandatory for grad, bonus for ugrads) (20 points)

Consider the following algorithms:

1. TT-ENTAILS?, AIMA Figure 7.10 page 221.
2. PL-RESOLUTION, AIMA Figure 7.13 page 228.
3. PL-FC-ENTAILS?, AIMA Figure 7.15 page 231.
4. DPLL-SATISFIABLE?, AIMA Figure 7.17 page 234.

5. WalkSAT, AIMA Figure 7.18 page 235.

For each of the above algorithms, carefully study the algorithm and explain how it operates by

- Clearly stating the input
- Providing the representation on which it operates
- Explaining when and why the algorithm stops
- Stating what mechanism the algorithm implements (for example by relating it to a known theorem.)

(4 points for each algorithm)

3 Using the inference rules for logic (10 points)

prove that "$\exists x\, Z(x)$ follows from the givens." Be sure to justify your steps by stating the inference rule used, along with the previous line(s) to which it was applied and the unifications used.

1.	$P(1)$	given
2.	$W(1) \wedge W(2) \wedge W(3)$	given
3.	$\forall x\, [P(x) \Rightarrow \neg R(x)]$	given
4.	$\forall x\, [Q(x) \vee R(x)]$	given
5.	$\forall x\, [(Q(x) \wedge W(x)) \Rightarrow Z(x)]$	given

4 Chapter 8, Exercise 4, Source: AIMA online site. (2 points)

Write down a logical sentence such that every world in which it is true contains exactly one object.

5 Chapter 8, Exercise 10, Source: AIMA online site. (19 points)

This exercise uses the function $MapColor$ and predicates $In(x, y)$, $Borders(x, y)$, and $Country(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

1. Paris and Marseilles are both in France.
   (a) $In(Paris \wedge Marseilles, France)$.
   (b) $In(Paris, France) \wedge In(Marseilles, France)$.
   (c) $In(Paris, France) \vee In(Marseilles, France)$.
2. There is a country that borders both Iraq and Pakistan.
   (a) $\exists\, c\ Country(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$.
   (b) $\exists\, c\ Country(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.
   (c) $\exists\, c\ Country(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.
   (d) $\exists\, c\ Border(Country(c), Iraq \wedge Pakistan)$.
3. All countries that border Ecuador are in South America.
   (a) $\forall\, c\, Country(c) \wedge Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$.
   (b) $\forall\, c\ Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$.
   (c) $\forall\, c\, [Country(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$.
   (d) $\forall\, c\, Country(c) \wedge Border(c, Ecuador) \wedge In(c, SouthAmerica)$.
4. No region in South America borders any region in Europe.
   (a) $\neg[\exists\, c, d\ In(c, SouthAmerica) \wedge In(d, Europe) \wedge Borders(c, d)]$.
   (b) $\forall\, c, d\, [In(c, SouthAmerica) \wedge In(d, Europe)] \Rightarrow \neg Borders(c, d)]$.
   (c) $\neg\forall\, c\ In(c, SouthAmerica) \Rightarrow \exists\, d\ In(d, Europe) \wedge \neg Borders(c, d)$.
   (d) $\forall\, c\ In(c, SouthAmerica) \Rightarrow \forall\, d\ In(d, Europe) \Rightarrow \neg Borders(c, d)$.
5. No two adjacent countries have the same map color.
   (a) $\forall\, x, y\, \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee \neg(MapColor(x) = MapColor(y))$.
   (b) $\forall\, x, y\, (Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg(x = y)) \Rightarrow \neg(MapColor(x) = MapColor(y))$.
   (c) $\forall\, x, y\ Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg(MapColor(x) = MapColor(y))$.
   (d) $\forall\, x, y\, (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow MapColor(x \neq y)$.

6 Chapter 8, Exercise 30, Source: AIMA online site. (12 points)

Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

1. Some students took French in spring 2001.

2. Every student who takes French passes it.
3. Only one student took Greek in spring 2001.
4. The best score in Greek is always higher than the best score in French.
5. Every person who buys a policy is smart.
6. No person buys an expensive policy.
7. There is an agent who sells policies only to people who are not insured.
8. There is a barber who shaves all men in town who do not shave themselves.
9. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.
10. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.
11. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.
12. All Greeks speak the same language. (Use $Speaks(x, l)$ to mean that person x speaks language l.)

7 Axioms in FOL (Adapted from AIMA, first edition) (15 points)

Using the following:

`Child(x,y)`, `Sibling(x,y)`, `Female(x)`, `Male(x)`, and `Spouse(x, y)`

- (10 points) Write axioms describing the predicates: `GrandChild`, `GreatGrandParent`, `Brother`, `Sister`, `Daughter`, `Son`, `Aunt`, `Uncle`, `BrotherInLaw`, `SisterInLaw`, and `FirstCousin`. We want these axioms to be definitions, so use $\Leftrightarrow$ instead of $\Rightarrow$.
- (5 points) Knowing that a second cousin is a child of one's parent first cousin, write the definition of a N^{th}`-cousin`, as a recursive expression in terms of the predicates defined above. Hint: Let N^{th}`-cousin` be a ternary predicate, that takes as input n, and two persons p_1 and p_2.

8 Chapter 9, Exercise 3, Source: AIMA online site. (3 points)

Suppose a knowledge base contains just one sentence, $\exists\, x\, AsHighAs(x, Everest)$. Which of the following are legitimate results of applying Existential Instantiation?

1. $AsHighAs(Everest, Everest)$.
2. $AsHighAs(Kilimanjaro, Everest)$.
3. $AsHighAs(Kilimanjaro, Everest) \wedge AsHighAs(BenNevis, Everest)$ (after two applications).

9 Chapter 9, Exercise 4, Source: AIMA online site. (4 points)

For each pair of atomic sentences, give the most general unifier if it exists:

1. $P(A, B, B)$, $P(x, y, z)$.
2. $Q(y, G(A, B))$, $Q(G(x, x), y)$.
3. $Older(Father(y), y)$, $Older(Father(x), John)$.
4. $Knows(Father(y), y)$, $Knows(x, x)$.

10 Chapter 9, Exercise 7, Source: AIMA online site. (12 points)

Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

1. Horses, cows, and pigs are mammals.
2. An offspring of a horse is a horse.
3. Bluebeard is a horse.
4. Bluebeard is Charlie's parent.
5. Offspring and parent are inverse relations.
6. Every mammal has a parent.

11 Chapter 9, Exercise 16, Source: AIMA online site. (12 points)

In this exercise, use the sentences you wrote in Chapter 9, Exercise 7 (Previous Question) to answer a question by using a backward-chaining algorithm.

1. Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $\exists\, h\, Horse(h)$, where clauses are matched in the order given.
2. What do you notice about this domain?
3. How many solutions for h actually follow from your sentences?
4. Can you think of a way to find all of them? (*Hint*: For this question, it is usefull to check the following paper Smith, D.E., Genesereth, M.R., and Ginsberg, M.L. (1986). *Controlling recursive inference.* Artificial Intelligence, Volume 30(3), pages 343–389.)

12 First-Order Logic (20 points)

Consider the following axioms:

1. Anyone who rides any Harley is a rough character.
2. Every biker rides [something that is] either a Harley or a BMW.
3. Anyone who rides any BMW is a yuppie.
4. Every yuppie is a lawyer.
5. Any nice girl does not date anyone who is a rough character.
6. Mary is a nice girl, and John is a biker.
7. (Conclusion) If John is not a lawyer, then Mary does not date John.

- Choose appropriate predicates to write the above axioms in first-order logic, clearly indicating the arguments and arity of each predicate: (2 points)
- Write each of the above axioms in first-order logic. Use scratch paper if necessary, and *neatly* report your results below. (10 points)

    1.
    2.
    3.
    4.
    5.
    6.

7.

- Transform each of the above sentences into a conjunctive normal form. Clearly state the Skolem functions and clearly number the statements. (4 points)
- Establish the conclusion using the axioms by applying refutation resolution. Clearly show the variable bindings at each step and clearly number the statements.

  (4 points)

  Negation of conclusion:

13 Unification (4 points)

What is the most general unifier of the following pairs of wff's? If none exists, report "fail." Assume that the capital letters are constants and the lowercase letters are variables.

1. $P(x, y, x, z)$ and $P(F(w), A, F(B), w)$
2. $Q(x, F(x), G(F(x)))$ and $Q(1, y, G(F(y)))$
3. Foo(x, y) and Foo(y, x)
4. Mother(x, y) and Mother$(y$,Father$(x))$

14 Unification and Resolution (2 points)

You are given the following pairs of clauses where upper case letters indicate constants and lower case letters indicate variables, functions, or predicates. Consider each pair independently of the others. In each pair, variables with the same name are meant to be the same variable. For each of the pairs, specify if the two clauses can be resolved. If yes, show the results of the unification process. If not, explain why.

1. $p(B, C, x, z, f(A, z, B))$ and $\neg p(y, z, y, C, w)$.
2. $r(f(y), y, x)$ and $\neg r(x, f(A), f(v))$.

15 Resolution and Refutation (5 points)

Use resolution and refutation to solve the problem below. Hint: First transform the givens into clausal form.
Given:

1. $\forall x(P(x) \Rightarrow Q(x))$
2. $\forall x(P(x) \Rightarrow (\forall y W(y))$
3. $\forall x \forall y((Q(x) \wedge W(y)) \Rightarrow S(x))$
4. $P(Mary)$

Show: $S(Mary)$

16 Translation into FOL (14 points)

Consider the set of all creatures. We will use the following predicates:

- $Insect(x)$.
- $Moth(x)$.
- $Dragonfly(x)$.
- $Spider(x)$.
- $Eats(x, y)$.
- $Wings(x, y)$: x has y wings.
- $Order(x)$. (Recall that zoologists classify creatures using kingdom, phylum, class, order, family, genus, and species.)

Translate the following sentences into logic. You may only use the identity predicate and the predicates and functions listed above. Try not to include more bugs in your logic than are required..

- Flik is an insect with 4 wings but is not a moth.
- Not all insects have 4 wings.
- All insects with 2 wings are in the same order.
- There are at least 3 difference orders.
- Moths and Dragonflies are insects but are not in the same order.
- All spiders eat insects.
- Some spiders eat only insects.

17 Inference in First-Order Logic: CNF and Resolution (60 points + 40 Bonus)

Choose *three* (3) of the exercices below, and for each of them, answer each of the following questions. The remaining two exercises are bonus, each worth 20 points.

- Choose appropriate predicates to write the above axioms in first-order logic.
- Write the axioms in First-Order Logic. Report your results neatly.
- Transform each of the first-order sentences into Conjunctive Normal Form. Clearly state the Skolem functions and clearly number the statements. Neatly report your results and provide as much detail as possible.
- Establish the conclusion using the axioms by applying *refutation resolution*. That is, negate the conclusion and prove the unsatisfiability of the set of clauses by resolution. Clearly show the variable bindings at each step and clearly number the statements.

17.1 Do Loons Eat Fish? (20 points)

Consider the following axioms.

1. Every bird sleeps in some tree.
2. Every loon is a bird, and every loon is aquatic.
3. Every tree in which any aquatic bird sleeps is beside some lake.
4. Anything that sleeps in anything that is beside any lake eats fish.
5. (Conclusion) Every loon eats fish.

17.2 Is there a conservative Austinite? (20 points)

Consider the following axioms:

1. Every Austinite who is not conservative loves some armadillo.
2. Anyone who wears maroon-and-white shirts is an Aggie.
3. Every Aggie loves every dog.
4. Nobody who loves every dog loves any armadillo.
5. Clem is an Austinite, and Clem wears maroon-and-white shirts.
6. (Conclusion) Is there a conservative Austinite?

17.3 Will Mary Date John? (20 points)

Consider the following axioms:

1. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
2. Every dog chases some rabbit.
3. Mary buys carrots by the bushel.
4. Anyone who owns a rabbit hates anything that chases any rabbit.
5. John owns a dog.
6. Someone who hates something owned by another person will not date that person.
7. (Conclusion) If Mary does not own a grocery store, she will not date John.

17.4 Drunk AI Students (20 points)

Consider the following axioms.

1. Everyone who feels warm either is drunk, or every costume they have is warm.
2. Every costume that is warm is furry.
3. Every AI student is a CS student.
4. Every AI student has some robot costume.
5. No robot costume is furry.
6. (Conclusion) If every CS student feels warm, then every AI student is drunk.

17.5 Brilliant CS Students (20 points)

Consider the following axioms:

1. Every student who makes good grades is brilliant or studies.
2. Every student who is a CS major has some roommate. [Make "roommate" a two-place predicate.]
3. Every student who has any roommate who likes to party goes to Sixth Street.
4. Anyone who goes to Sixth Street does not study.
5. (Conclusion) If every roommate of every CS major likes to party, then every student who is a CS major and makes good grades is brilliant.