Title: First-Order Logic
AIMA: Chapter 8 (Sections 8.1 and 8.2)

Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence
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Outline

• First-order logic:
  – basic elements
  – syntax
  – semantics

• Examples
Pros and cons of propositional logic

- Propositional logic is declarative: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
  (unlike most data structures and databases)
- Propositional logic is compositional:
  meaning of $B_{1,1} \land P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent
  (unlike natural language, where meaning depends on context)
- but...
  Propositional logic has very limited expressive power
  E.g., cannot say “pits cause breezes in adjacent squares” except by writing one sentence for each square
Propositional Logic

- is simple
- illustrates important points: model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness:
  → In PL, world contains facts

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)
First Order Logic

→ FOL provides more "primitives" to express knowledge:
   — objects (identity & properties)
   — relations among objects (including functions)

**Objects:** people, houses, numbers, Einstein, Huskers, event, ..

**Properties:** smart, nice, large, intelligent, loved, occurred, ..

**Relations:** brother-of, bigger-than, part-of, occurred-after,..

**Functions:** father-of, best-friend, double-of, ..

**Examples:**
   (objects? function? relation? property?)
   — one plus two equals four
   — squares neighboring the wumpus are smelly
Logic

_Attracts:_ mathematicians, philosophers and AI people

_Arguments:_
— allows to represent the world and reason about it
— expresses anything that can be programmed

_Non-committal to:_
— symbols could be objects or relations
  (e.g., King(Gustave), King(Sweden, Gustave), Merciless(King))
— classes, categories, time, events, uncertainty

_.. but amenable to_ extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

→ Some people think FOL *is* the language of AI
ture/false? donno :—( but it will remain around for some time..
Types of logic

Logics are characterized by what they commit to as “primitives”

Ontological commitment:
what exists—facts? objects? time? beliefs?

Epistemological commitment:
what states of knowledge?

<table>
<thead>
<tr>
<th>Language</th>
<th>Ontological Commitment</th>
<th>Epistemological Commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>First-order logic</td>
<td>facts, objects, relations</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Temporal logic</td>
<td>facts, objects, relations, times</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Probability theory</td>
<td>facts</td>
<td>degree of belief 0…1</td>
</tr>
<tr>
<td>Fuzzy logic</td>
<td>degree of truth</td>
<td>degree of belief 0…1</td>
</tr>
</tbody>
</table>

Higher-Order Logic: views relations and functions of FOL as objects
Syntax of FOL: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: $x$, $y$, etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation) Father-of, Square-root, LeftLeg, etc.
- Quantifiers $\forall$, $\exists$
- Connectives: $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$
- (Sometimes) equality $=$

Predicates and functions can have any arity (number of arguments)
Basic elements in FOL (i.e., the grammar)

In propositional logic, every expression is a sentence

In FOL,

- Terms

- Sentences:
  - atomic sentences
  - complex sentences

- Quantifiers:
  - Universal quantifier
  - Existential quantifier
Term

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

\[
\text{Term} = \text{function}(\text{term}_1, \ldots, \text{term}_n)
\]

or constant or variable

— ground term: term with no variable
Atomic sentences

state facts

built with terms and predicate symbols

Atomic sentence = \textit{predicate}(\textit{term}_1, \ldots, \textit{term}_n)

or \textit{term}_1 = \textit{term}_2

Examples:

- Brother (Richard, John)
- Married (FatherOf(Richard), MotherOf(John))
Complex Sentences

built with atomic sentences and logical connectives

$\neg S$

$S_1 \land S_2$

$S_1 \lor S_2$

$S_1 \Rightarrow S_2$

$S_1 \Leftrightarrow S_2$

Examples:

Sibling(KingJohn, Richard) $\Rightarrow$ Sibling(Richard, KingJohn)

$>(1, 2) \lor \leq(1, 2)$

$>(1, 2) \land \neg>(1, 2)$
**Truth in first-order logic:** Semantic

Sentences are true with respect to a model and an interpretation. Model contains objects and relations among them.

Interpretation specifies referents for:

- **constant symbols → objects**
- **predicate symbols → relations**
- **function symbols → functional relations**

An atomic sentence `predicate(term_1, \ldots, term_n)` is true iff the objects referred to by `term_1, \ldots, term_n` are in the relation referred to by `predicate`.
Model in FOL: example

The **domain** of a model is the set of objects it contains:

five objects

**Intended interpretation:** Richard refers Richard the Lion Heart, John refers to Evil King John, Brother refers to brotherhood relation, etc.
Models for FOL: Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements \( n \) from 1 to \( \infty \)
   For each \( k \)-ary predicate \( P_k \) in the vocabulary
      For each possible \( k \)-ary relation on \( n \) objects
         For each constant symbol \( C \) in the vocabulary
            For each choice of referent for \( C \) from \( n \) objects . . .

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded
\[ \rightarrow \] Checking entailment by enumerating is not an option
Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things
Universal quantification

∀ ⟨variables⟩ ⟨sentence⟩

Example: all dogs like bones ∀ x Dog(x) ⇒ LikeBones(x)

x = Indy is a dog x = Indiana Jones is a person

∀ x P is equivalent to the conjunction of instantiations of P

Dog(Indy) ⇒ LikeBones(Indy)
∧ Dog(Rebel) ⇒ LikeBones(Rebel)
∧ Dog(KingJohn) ⇒ LikeBones(KingJohn)
∧ ...

Typically: ⇒ is the main connective with ∀

Common mistake: using ∧ as the main connective with ∀

Example: ∀ x Dog(x) ∧ LikeBones(x)

all objects in the world are dogs, and all like bones
Existential quantification

$\exists \langle \text{variables} \rangle \langle \text{sentence} \rangle$

**Example:** some student will talk at the TechFair

$\exists x \text{Student}(x) \land \text{TalksAtTechFair}(x)$

Pat, Leslie, Chris are students

$\exists x \ P$ is equivalent to the disjunction of instantiations of $P$

\[
\begin{align*}
\text{Student}(\text{Pat}) \land \text{TalksAtTechFair}(\text{Pat}) \\
\lor \text{Student}(\text{Leslie}) \land \text{TalksAtTechFair}(\text{Leslie}) \\
\lor \text{Student}(\text{Chris}) \land \text{TalksAtTechFair}(\text{Chris}) \\
\lor \ldots
\end{align*}
\]

**Typically:** $\land$ is the main connective with $\exists$

**Common mistake:** using $\Rightarrow$ as the main connective with $\exists$

$\exists x \text{Student}(x) \Rightarrow \text{TalksAtTechFair}(x)$

is true if there is anyone who is not Student
Properties of quantifiers (I)

∀x ∀y is the same as ∀y ∀x

∃x ∃y is the same as ∃y ∃x

∃x ∀y is not the same as ∀y ∃x

∃x ∀y Loves(x, y)

“There is a person who loves everyone in the world”

∀y ∃x Loves(x, y)

“Everyone in the world is loved by at least one person”

Quantifier duality: each can be expressed using the other

∀x Likes(x, IceCream) → ∃x ¬Likes(x, IceCream)

∃x Likes(x, Broccoli) → ∀x ¬Likes(x, Broccoli)

Parsimony principal: ∀, ¬, and ⇒ are sufficient
Properties of quantifiers (II)

Nested quantifier:
\( \forall x (\exists y (P(x, y))) \):

- every object in the world has a particular property, which is the property to be related to some object by the relation \( P \)

\( \exists x (\forall y (P(x, y))) \):

- there is some object in the world that has a particular property, which is the property to be related to every object by the relation \( P \)

**Lexical scoping:** \( \forall x [\text{Cat}(x) \lor \exists x \text{Brother}(\text{Richard}, x)] \)

**Well-formed formulas (WFF):** (kind of correct spelling)

- every variable must be introduced by a quantifier

\( \forall x P(y) \) is not a WFF
Examples

Brothers are siblings

“Sibling” is symmetric

One’s mother is one’s female parent

A first cousin is a child of a parent’s sibling
Examples

\[ \forall x, y \ \text{Brother}(x, y) \implies \text{Sibling}(x, y) \]

\[ \forall x, y \ \text{Sibling}(x, y) \implies \text{Sibling}(y, x) \]

\[ \forall x, y \ \text{Mother}(x, y) \implies (\text{Female}(x) \land \text{Parent}(x, y)) \]

\[ \forall x, y \ \text{FirstCousin}(x, y) \iff \exists a, b \ \text{Parent}(a, x) \land \text{Sibling}(a, b) \land \text{Parent}(b, y) \]
Tricky example

Someone is loved by everyone
\[ \exists x \forall y \, Loves(y, x) \]

Someone with red-hair is loved by everyone
\[ \exists x \forall y \, Redhair(x) \land Loves(y, x) \]

Alternatively:
\[ \exists x \, Person(x) \land Redhair(x) \land (\forall y \, Person(y) \Rightarrow Loves(y, x)) \]
Equality

\( \text{term}_1 = \text{term}_2 \) is true under a given interpretation if and only if \( \text{term}_1 \) and \( \text{term}_2 \) refer to the same object

Examples

- Father(John) = Henry
- \( 1 = 2 \) is satisfiable
- \( 2 = 2 \) is valid
- Useful to distinguish two objects:
  - Definition of (full) Sibling in terms of Parent:
    \[
    \forall x, y \ Sibling(x, y) \iff [\neg(x = y) \land \exists m, f \neg(m = f) \land Parent(m, x) \land Parent(f, x) \land Parent(m, y) \land Parent(f, y)]
    \]
    Spot has at least two sisters: ...

AIMA, Exercise 8.4. Write: “All Germans speak the same languages,” where \( \text{Speaks}(x, l) \) means that person \( x \) speaks language \( l \).
**Knowledge representation** (KR)

**Domain**: a section of the world about which we wish to express some knowledge

**Example**: Family relations (kinship):
- **Objects**: people
- **Properties**: gender, married, divorced, single, widowed
- **Relations**: parenthood, brotherhood, marriage..

**Unary predicates**: Male, Female

**Binary relations**: Parent, Sibling, Brother, Child, etc.

**Functions**: Mother, Father

\[ \forall m, c, Mother(c) = m \iff Female(m) \land Parent(m, c) \]
In Logic (informally)

- Basic facts: axioms
- Derived facts: theorems

**Independent axiom**

an axiom that cannot be derived from the rest

→ Goal of mathematicians: find the minimal set of independent axioms

In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK
Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t = 5$:

$\text{Tell}(KB, \text{Percept}([\text{Smell}, \text{Breeze}, \text{None}], 5))$
$\text{Ask}(KB, \exists a \text{Action}(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: Yes, $\{a/\text{Shoot}\} \leftarrow \text{substitution}$ (binding list)

Given a sentence $S$ and a substitution $\sigma$,
$S\sigma$ denotes the result of plugging $\sigma$ into $S$; e.g.,
$S = \text{Smarter}(x, y)$
$\sigma = \{x/\text{Hillary}, y/\text{Bill}\}$
$S\sigma = \text{Smarter}(\text{Hillary}, \text{Bill})$

$\text{Ask}(KB, S)$ returns some/all $\sigma$ such that $KB \models S\sigma$
Prepare for next lecture: AIMA, Exercise 8.24, page 319

Takes($x, c, s$): student $x$ takes course $c$ in semester $s$

Passes($x, c, s$): student $x$ passes course $c$ in semester $s$

Score($x, c, s$): the score obtained by student $x$ in course $c$ in semester $s$

$x > y$: $x$ is greater than $y$

$F$ and $G$: specific French and Greek courses

Buys($x, y, z$): $x$ buys $y$ from $z$

Sells($x, y, z$): $x$ sells $y$ from $z$

Shaves($x, y$): person $x$ shaves person $y$

Born($x, c$): person $x$ is born in country $c$

Parent($x, y$): person $x$ is parent of person $y$

Citizen($x, c, r$): person $x$ is citizen of country $c$ for reason $r$

Resident($x, c$): person $x$ is resident of country $c$ of person $y$

Fools($x, y, t$): person $x$ fools person $y$ at time $t$

Student ($x$), Person($x$), Man($x$), Barber($x$), Expensive($x$), Agent($x$), Insured($x$), Smart($x$), Politician($x$),
AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986
head of AI at AT&T Labs-Research
Program co-chair of AAAI-2000
Professor at University of Washington, Seattle

Founding Director of Institute for Data Science and Professor at University of Rochester