Title: Adverserial Search
AIMA: Chapter 5 (Sections 5.1, 5.2 and 5.3)
Outline

• Introduction
• Minimax algorithm
• Alpha-beta pruning
Context

- In an MAS, agents affect each other’s welfare
- Environment can be cooperative or competitive
- Competitive environments yield adversarial search problems (games)
- Approaches: mathematical game theory and AI games
Game theory vs. AI

- AI games: fully observable, deterministic environments, players alternate, utility values are equal (draw) or opposite (winner/loser)
  In vocabulary of game theory: deterministic, turn-taking, two-player, zero-sum games of perfect information

- Games are attractive to AI: states simple to represent, agents restricted to a small number of actions, outcome defined by simple rules
  Not croquet or ice hockey, but typically board games
  Exception: Soccer (Robocup www.robocup.org/)
Board game playing: an appealing target of AI research

Board game: Chess (since early AI), Othello, Go, Backgammon, etc.

- Easy to represent
- Fairly small numbers of well-defined actions
- Environment fairly accessible
- Good abstraction of an enemy, w/o real-life (or war) risks :—)

But also: Bridge, ping-pong, etc.
Characteristics

- ‘Unpredictable’ opponent: contingency problem (interleaves search and execution)

- Not the usual type of ‘uncertainty’:
  no randomness/no missing information (such as in traffic)
  but, the moves of the opponent expectedly non benign

- Challenges:
  - huge branching factor
  - large solution space
  - Computing optimal solution is infeasible
  - Yet, decisions must be made. Forget A*...
Discussion

• What are the theoretically best moves?

• Techniques for choosing a good move when time is tight
  √ Pruning: ignore irrelevant portions of the search space
  × Evaluation function: approximate the true utility of a state
    without doing search
Two-person Games

- 2 player: Min and Max
- Max moves first
- Players alternate until end of game
- Gain awarded to player/penalty give to loser

Game as a search problem:

- Initial state: board position & indication whose turn it is
- Successor function: defining legal moves a player can take
  Returns \{(move, state)\}*
- Terminal test: determining when game is over
  states satisfy the test: terminal states
- Utility function (a.k.a. payoff function): numerical value for outcome e.g., Chess: win=1, loss=-1, draw=0
Usual search

Max finds a sequence of operators yielding a terminal goal scoring winner according to the utility function.

Game search

- Min actions are significant
  Max must find a strategy to win regardless of what Min does:
  $$\rightarrow$$ correct action for Max for each action of Min
- Need to approximate (no time to envisage all possibilities difficulty): a huge state space, an even more huge search space
  \[10^{40}\text{ different legal positions}\]
  \[\text{Average branching factor}=35, \text{ 50 moves/player}= 35^{100}\]
- Performance in terms of time is very important
Example: Tic-Tac-Toe

Max has 9 alternative moves

Terminal states’ utility: Max wins=1, Max loses = -1, Draw = 0
**Example:** 2-ply game tree

Max’s actions: $a_1$, $a_2$, $a_3$

Min’s actions: $b_1$, $b_2$, $b_3$

Minimax algorithm determines the optimal strategy for Max → decides which is the best move
Minimax algorithm

- Generate the whole tree, down to the leaves
- Compute utility of each terminal state
- Iteratively, from the leaves up to the root, use utility of nodes at depth \( d \) to compute utility of nodes at depth \((d - 1)\):
  
  \[
  \begin{aligned}
  \text{MIN 'row': minimum of children} \\
  \text{MAX 'row': maximum of children}
  \end{aligned}
  \]

**Minimax-Value** (\( n \))

\[
\begin{aligned}
\text{Utility}(n) & \quad \text{if } n \text{ is a terminal node} \\
\max_{s \in \text{Succ}(n)} \text{Minimax-Value}(s) & \quad \text{if } n \text{ is a Max node} \\
\min_{s \in \text{Succ}(n)} \text{Minimax-Value}(s) & \quad \text{if } n \text{ is a Min node}
\end{aligned}
\]
Minimax decision

- MAX’s decision: **minimax decision** maximizes utility under the assumption that the opponent will play perfectly to his/her own advantage

- Minimax decision maximizes the worst-case outcome for Max (which otherwise is guaranteed to do better)

- If opponent is sub-optimal, other strategies may reach better outcome better than the minimax decision
Minimax algorithm: Properties

- $m$ maximum depth
  $b$ legal moves

- Using Depth-first search, space requirement is:
  $O(bm)$: if generating all successors at once
  $O(m)$: if considering successors one at a time

- Time complexity $O(b^m)$

Real games: time cost totally unacceptable
Multiple players games

\( \text{UTILITY}(n) \) becomes a vector of the size of the number of players

For each node, the vector gives the utility of the state for each player to move

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{A}
\end{array}
\]

A

(1, 2, 6) (4, 2, 3) (6, 1, 2) (7, 4, 1) (5, 1, 1) (1, 5, 2) (7, 7, 1) (5, 4, 5)

B

(1, 2, 6)

C

X

(1, 2, 6) (6, 1, 2) (1, 5, 2) (5, 4, 5)
Alliance formation in multiple players games

How about alliances?

• A and B in weak positions, but C in strong position
  A and B make an alliance to attack C (rather than each other)
  → Collaboration emerges from purely selfish behavior!

• Alliances can be done and undone (careful for social stigma!)

• When a two-player game is not zero-sum, players may end up automatically making alliances (for example when the terminal state maximizes utility of both players)
Alpha-beta pruning

- Minimax requires computing all terminal nodes: unacceptable

- Do we really need to do compute utility of all terminal nodes?  
  ... No, says John McCarthy in 1956:

  It is possible to compute the correct minimax decision without looking at every node in the tree, and yet get the correct decision

- Use pruning (eliminating useless branches in a tree)
**Example** of alpha-beta pruning

(a) \([-\infty, +\infty]\)  
\([-\infty, 3]\)  
3  
(b) \([-\infty, +\infty]\)  
\([-\infty, 3]\)  
3 12  
(c) \([3, +\infty]\)  
\([3, 3]\)  
3 12 8  
(d) \([3, +\infty]\)  
\([3, 3]\)  
3 12 8 2  
(e) \([3, 14]\)  
\([3, 3]\)  
\([-\infty, 2]\)  
\([-\infty, 14]\)  
3 12 8 2 14  
(f) \([3, 3]\)  
\([3, 3]\)  
\([-\infty, 2]\)  
\([2, 2]\)  
3 12 8 2 14 5 2

Try 14, 5, 2, 6 below D
**General principal** of Alpha-beta pruning

If Player has a better choice \( m \) at

\[
\begin{cases}
- \text{a parent node of } n \\
- \text{any choice point further up}
\end{cases}
\]

\( n \) will never be reached in **actual play**

Once we have found enough about \( n \) \((e.g.,\) through one of it descendants\), we can prune it \((i.e.,\) discard all its remaining descendants\)
**Mechanism** of Alpha-beta pruning

\(\alpha\): value of best choice so far for MAX, (maximum)
\(\beta\): value of best choice so far for MIN, (minimum)

**Alpha-beta search:**
- updates the value of \(\alpha, \beta\) as it goes along
- prunes a subtree as soon as its worse then current \(\alpha\) or \(\beta\)
Effectiveness of pruning depends on the order of new nodes examined.

(a) \([-\infty, +\infty]\) 
(b) \([-\infty, 3]\) 
(c) \([3, +\infty]\) 
(d) \([3, +\infty]\) 
(e) \([3, 14]\) 
(f) \([3, 2]\)
**Savings** in terms of cost

- **Ideal case:**
  Alpha-beta examines $O(b^{d/2})$ nodes (vs. Minimax: $O(b^d)$)
  $\rightarrow$ Effective branching factor $\sqrt{b}$ (vs. Minimax: $b$)

- **Successors ordered randomly:**
  $b > 1000$, asymptotic complexity is $O((b/\log b)^d)$
  $b$ reasonable, asymptotic complexity is $O(b^{3d/4})$

- **Practically:** Fairly simple heuristics work (fairly) well