CSCE476/876 **Fall 2020**

Homework 6

Assigned on: Friday, October 16, 2020.

Due: Friday, October 23, 2020.

Points: 120 points + up to 20 bonus

Contents

Alert: If you submit your homework handwritten, it must be *absolutely neat* or it *will not* be corrected. If you type your homework (preferable), submit using webhandin.

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1 SAT Modeling (15 Points)

For each of the following scenarios, write a CNF formula to describe the scenario and complete the following four steps:

1. First state the propositions and what they represent.
2. State the sentence.
3. Explain the meaning of the clauses.
4. Is the sentence satisfiable? Explain why or why not.

1.1 Scenario A (5 Points)

Write a CNF formula to model the following scenario and complete the four steps from above:

1. There are four choices of desserts: ice cream, fruit bowl, cake, pie.
2. Exactly one dessert must be selected (i.e., one and only one).

1.2 Scenario B (5 Points)

Write a CNF formula to model the following scenario and complete the four steps from above:

1. Damon, Enrique, and Lois need to complete a paper and a presentation for a class.
2. To complete each task, they need to select a day to meet during the week (Mon, Tue, Wed, Thu, Fri).
3. Damon cannot meet on Monday. Further, he wants to complete the paper before the presentation and not both on the same day.
4. Enrique can meet any day but cannot meet on two consecutive days.
5. Lois wants to complete the presentation on or before Wednesday.

1.3 Scenario C (5 Points)

Write a CNF formula to model the following scenario and complete the four steps from above:

1. The four states (NE, IA, KS, MO) on the map shown in Figure **??** must be colored using three colors: red, green, and blue.

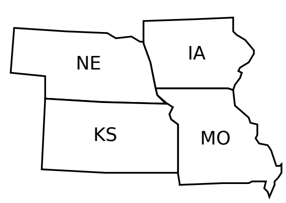

Figure 1: Four states (NE, IA, KS, MO)

2. Each state must be colored with exactly one color.
3. Adjacent states (i.e., states sharing a border line) cannot have the same color.

2 AIMA, Exercise 7.1, page 279. (16 points)

3 AIMA, Exercise 7.2, page 280. (5 points)

4 AIMA, Exercise 7.7, page 281. (6 points)

5 Truth Tables (8 points)

Use truth tables to show that each of the following is a tautology.

1. $(p \wedge q) \rightarrow \neg(\neg p \vee \neg q)$
2. $[Mary \wedge (Mary \rightarrow Susy)] \rightarrow Susy$
3. $\alpha \rightarrow [\beta \rightarrow (\alpha \wedge \beta)]$
4. $(a \rightarrow b) \rightarrow [(b \rightarrow c) \rightarrow (a \rightarrow c)]$

6 AIMA, Exercise 7.10, page 281. (16 points)

Only b, c, d, e, f, and g.

7 Logical Equivalences (8 points)

Using a method of your choice, verify:

1. $(\alpha \rightarrow \beta) \equiv (\neg\beta \rightarrow \neg\alpha)$ contraposition
2. $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ de Morgan
3. $(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \gamma) \vee (\alpha \wedge \beta))$ distributivity of $\wedge$ over $\vee$

8 AIMA, Exercise 7.22, page 284. (18 points + 20 bonus)

Parts a, b, and c are required. Parts d, e, and f are bonus.

9 Proofs (28 points)

Give the explantions of each step if the steps are given, and give both the explanation and step if they are not.

- If $q \wedge (r \wedge p), t \rightarrow v, v \rightarrow \neg p$, then $\neg t \wedge r$.

Proof	**Explanations**
1. $q \wedge (r \wedge p)$	Given
2. $t \rightarrow v$	Given
3. $v \rightarrow \neg p$	Given
4. $t \rightarrow \neg p$	
5. $(r \wedge p)$	
6. r	
7. p	

8. $\neg\neg p$
9. $\neg t$
10. $\neg t \wedge r$

- If $p \to (q \wedge r), q \to s$, and $r \to t$, then $p \to (s \wedge t)$.

  Proof **Explanations**

  1.
  2.
  3.
  4.
  5.
  6.
  7.

- **Prove by contradiction.**

  If $\neg(\neg p \wedge q), p \to (\neg t \vee r), q$, and t, then r.

  Proof **Explanations**

  1. $\neg(\neg p \wedge q)$ Given
  2. $p \to (\neg t \vee r)$ Given
  3. q Given
  4. t Given
  5. $\neg r$ Negation of Conclusion
  6.
  7.
  8.
  9.
  10.
  11.
  12.