

Title: Solving Problems by Searching

AIMA: Chapter 3 (Sections 3.4)

Introduction to Artificial Intelligence

CSCE 476-876, Fall 2019

**URL:** [www.cse.unl.edu/~cse476](http://www.cse.unl.edu/~cse476) **URL:**  
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Berthe Y. Choueiry (Shu-we-ri)

[choueiry@cse.unl.edu](mailto:choueiry@cse.unl.edu), (402)472-5444

```
function GENERAL-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

Essence of search: which node to expand first?

→ search strategy

A strategy is defined by picking the *order of node expansion*

## Types of Search

**Uninformed:** use only information available in problem definition

**Heuristic:** exploits some knowledge of the domain

## Uninformed search strategies

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening depth-first search
6. Bidirectional search

## Search strategies

### Criteria for evaluating search:

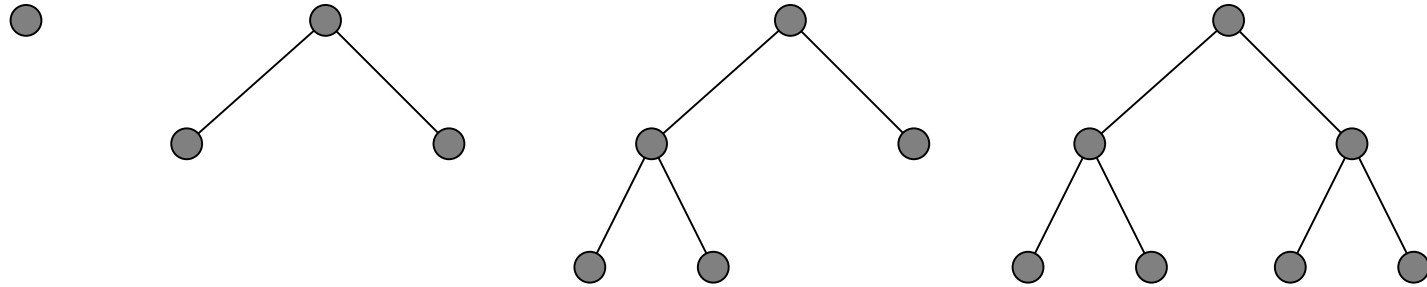
1. Completeness: does it always find a solution if one exists?
2. Time complexity: number of nodes generated/expanded
3. Space complexity: maximum number of nodes in memory
4. Optimality: does it always find a least-cost solution?

### Time/space complexity measured in terms of:

- $b$ : maximum branching factor of the search tree
- $d$ : depth of the least-cost solution
- $m$ : maximum depth of the search space (may be  $\infty$ )

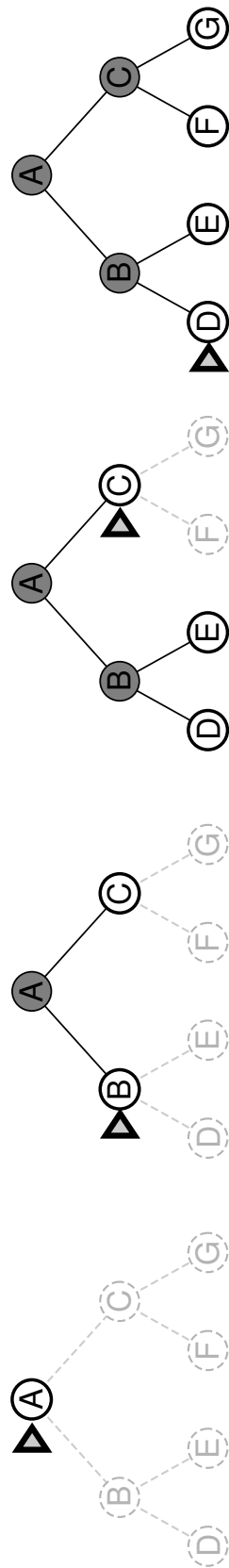
## Breadth-first search (I)

- Expand root node
- Expand all children of root
- Expand *each* child of root
- Expand successors of each child of root, etc.



- Expands nodes at depth  $d$  before nodes at depth  $d + 1$
- Systematically considers all paths length 1, then length 2, etc.
- Implement: put successors at end of queue.. FIFO

# Breadth-first search (2)



## Breadth-first search (3)

→ One solution?

→ Many solutions? Finds shallowest goal first

1. Complete? Yes, if  $b$  is finite
2. Optimal? provided cost increases monotonically with depth, not in general (e.g., actions have same cost)

3. Time?  $1 + b + b^2 + b^3 + \dots + b^d + b(b^d - 1) = O(b^{d+1})$

$$O(b^{d+1}) \left\{ \begin{array}{l} \text{branching factor } b \\ \text{depth } d \end{array} \right.$$

4. Space? same,  $O(b^{d+1})$ , keeps every node in memory, big problem

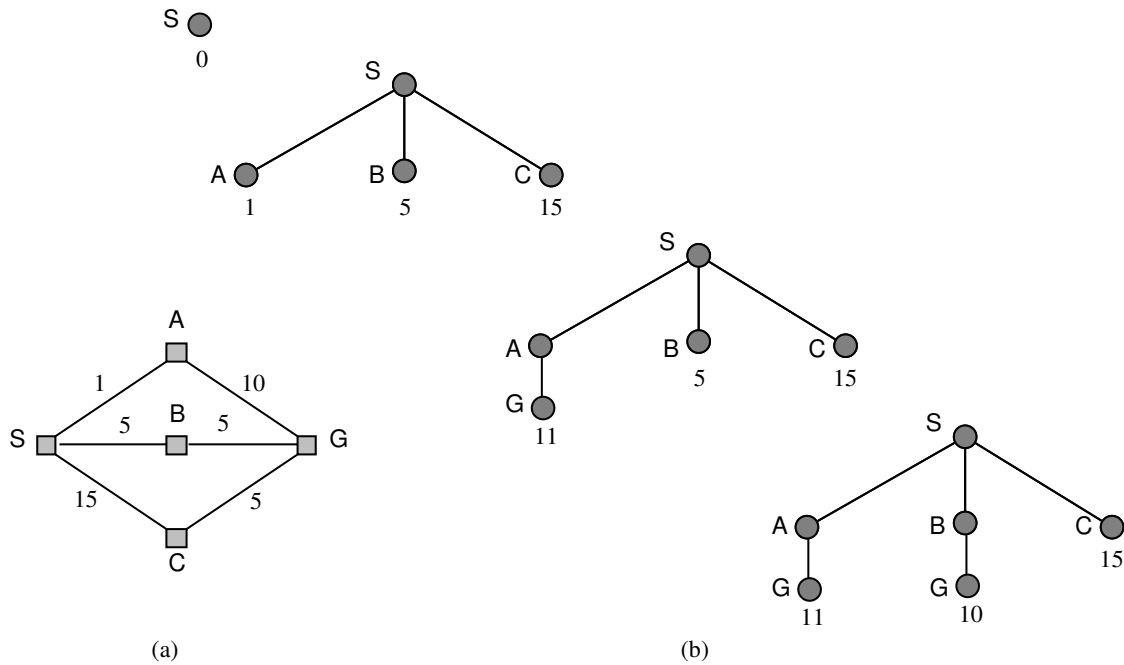
can easily generate nodes at 10MB/sec so 24hrs = 860GB

# Uniform-cost search (I)

- Breadth-first does not consider path cost  $g(x)$
- Uniform-cost expands first lowest-cost node on the fringe
- Implement: sort queue in decreasing cost order

When  $g(x) = \text{Depth}(x) \rightarrow \text{Breadth-first} \equiv \text{Uniform-cost}$

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## Uniform-cost search (2)

1. Complete?

Yes, if cost  $\geq \epsilon$

2. Optimal?

If the cost is a monotonically increasing function

When cost is added up along path, an operator's cost .....?

3. Time?

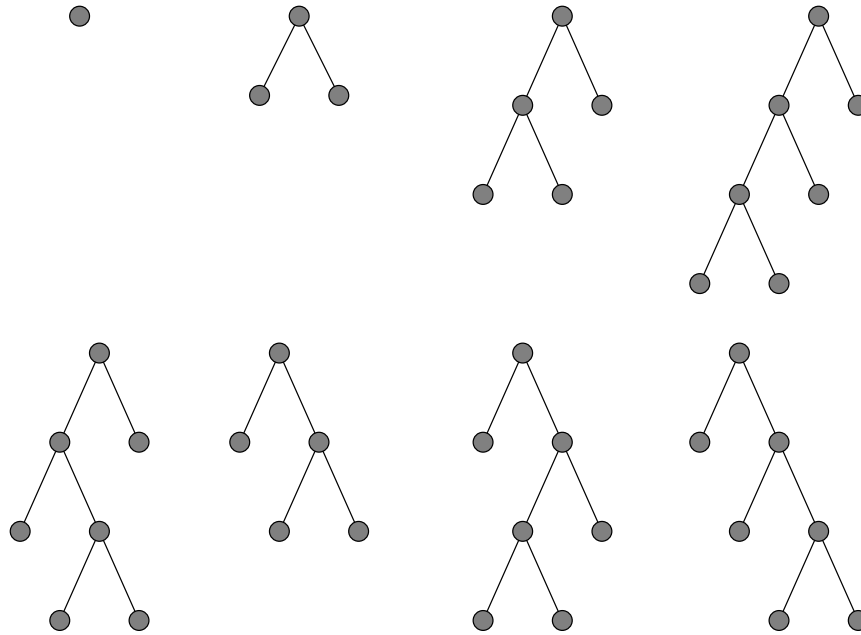
# of nodes with  $g \leq$  cost of optimal solution,  $O(b^{\lceil C^*/\epsilon \rceil})$   
where  $C^*$  is the cost of the optimal solution

4. Space?

# of nodes with  $g \leq$  cost of optimal solution,  $O(b^{\lceil C^*/\epsilon \rceil})$

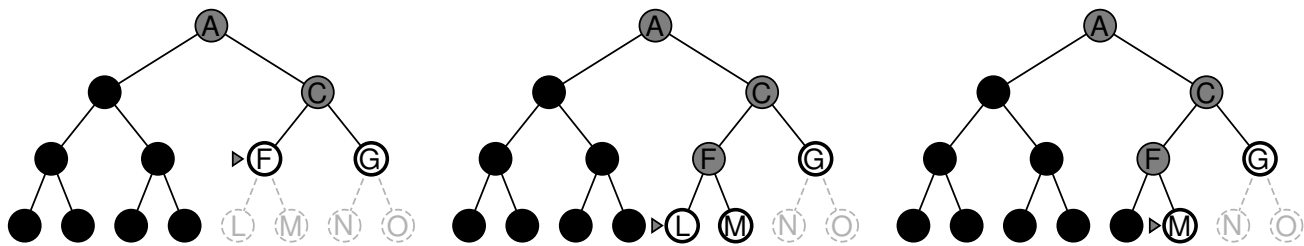
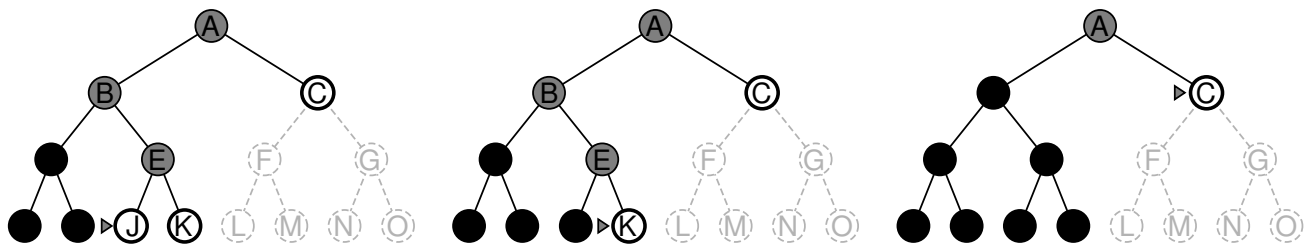
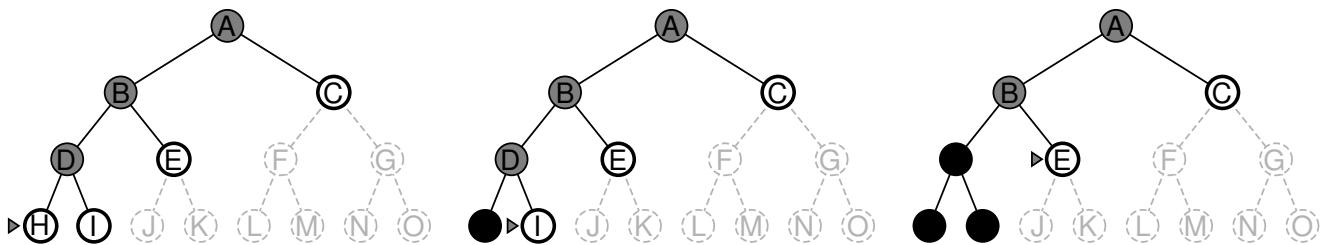
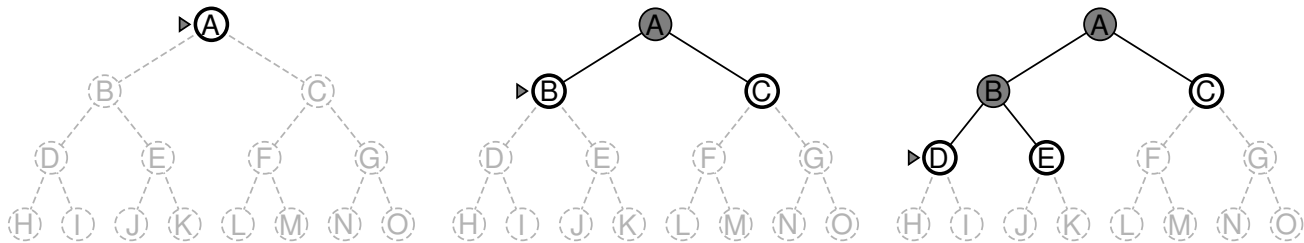
## Depth-first search (I)

- Expands nodes at deepest level in tree
- When dead-end, goes back to shallower levels
- Implement: put successors at front of queue.. LIFO



- Little memory: path and unexpanded nodes
- For  $b$ : branching factor,  $m$ : maximum depth, space .....

# Depth-first search (2)



## Depth-first search (3)

Time complexity:

We may need to expand all paths,  $O(b^m)$

When there are many solutions, DFS may be quicker than BFS

When  $m$  is big, much larger than  $d$ ,  $\infty$  (deep, loops), .. troubles

→ Major drawback of DFS: going deep where there is no solution..

### Properties:

1. Complete? Not in infinite spaces, complete in finite spaces

2. Optimal?

3. Time?  $O(b^m)$

Woow..

terrible if  $m$  is much larger than  $d$ , but if solutions are dense,  
may be much faster than breadth-first

4. Space?  $O(bm)$ , linear!

Woow..

## Depth-limited search (I)

→ DFS is going too deep, put a threshold on depth!

For instance, 20 cities on map for Romania, any node deeper than 19 is cycling. Don't expand deeper!

→ Implement: nodes at depth  $l$  have no successor

### Properties:

1. Complete?
2. Optimal?
3. Time? (given  $l$  depth limit)
4. Space? (given  $l$  depth limit)

**Problem:** how to choose  $l$ ?

# Iterative-deepening search (I)

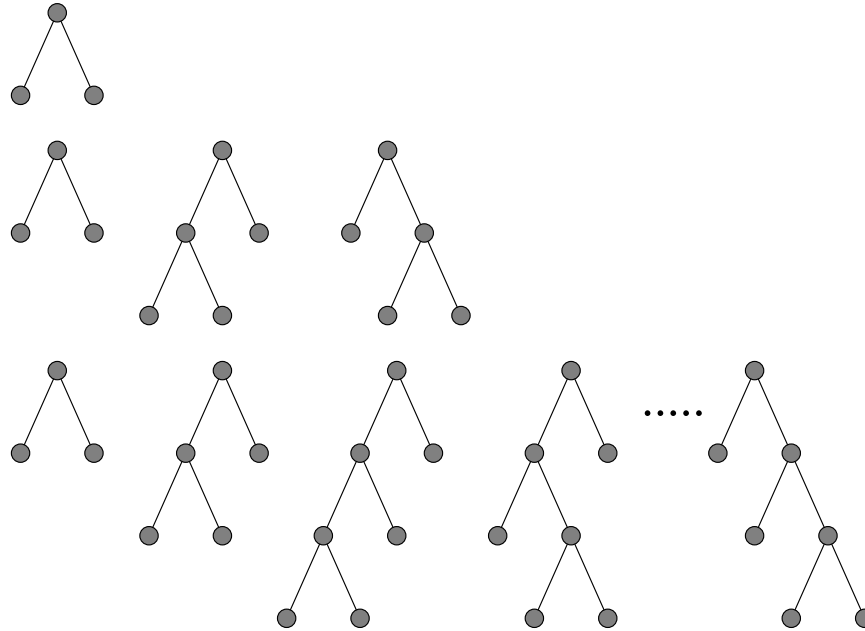
- DLS with depth = 0
- DLS with depth = 1
- DLS with depth = 2
- DLS with depth = 3...

Limit = 0 ●

Limit = 1 ●

Limit = 2 ●

Limit = 3 ●



→ Combines benefits of DFS and BFS

# Iterative-deepening search (2)

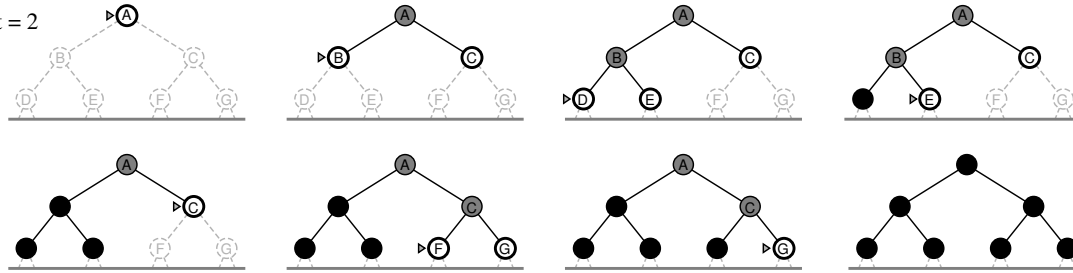
Limit = 0



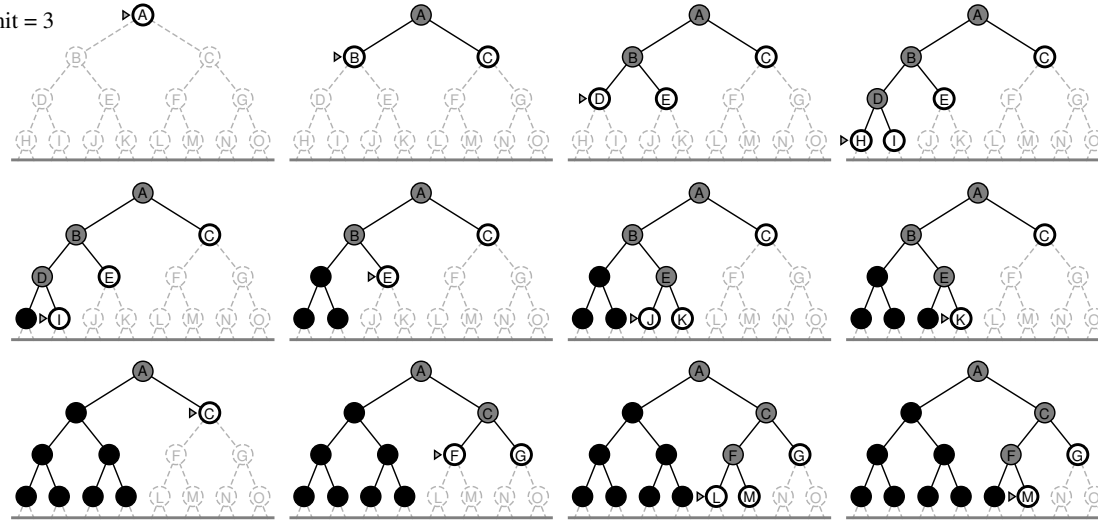
Limit = 1



Limit = 2



Limit = 3



## Iterative-deepening search (3)

→ combines benefits of DFS and BFS

### Properties:

1. Time?  $(d + 1).b^0 + (d).b + (d - 1).b^2 + \dots + 1.b^d = O(b^d)$
2. Space?  $O(bd)$ , like DFS
3. Complete? like BFS
4. Optimal? like BFS (if step cost = 1)



## Iterative-deepening search (4)

→ Some nodes are expanded several times, wasteful?

$$N(\text{BFS}) = b + b^2 + b^3 + \dots + b^d + (b^{d+1} - b)$$

$$N(\text{IDS}) = (d)b + (d-1)b^2 + \dots + (1)b^d$$

Numerical comparison for  $b = 10$  and  $d = 5$ :

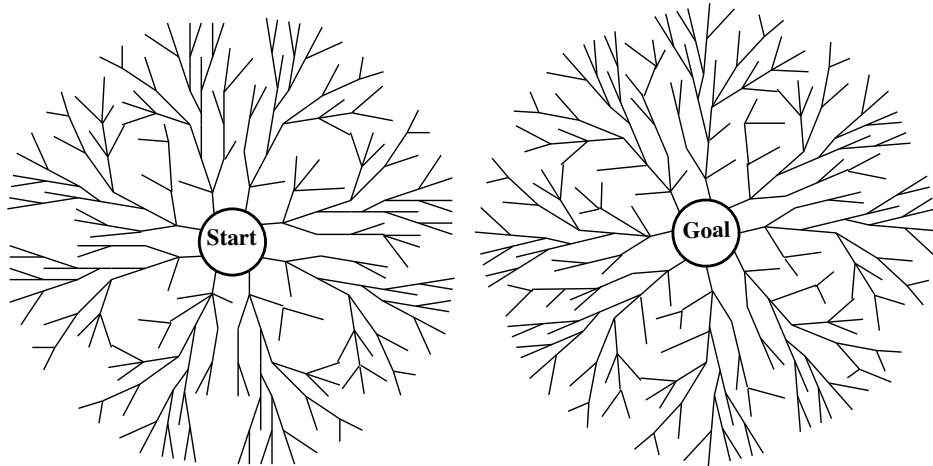
$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$

$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

→ IDS is preferred when search space is large and depth unknown

## Bidirectional search (I)

→ Given initial state and the goal state, start search from both ends and meet in the middle



→ Assume same  $b$  branching factor,  $\exists$  solution at depth  $d$ , time:

$$O(2b^{d/2}) = O(b^{d/2})$$

$$b = 10, d = 6, \text{DFS} = 1,111,111 \text{ nodes, BDS} = 2,222 \text{ nodes!}$$

## Bidirectional search (2)

**In practice** :—(

- Need to define predecessor operators to search backwards  
If operator are invertible, no problem
- What if  $\exists$  many goals (set state)?  
do as for multiple-state search
- need to check the 2 fringes to see how they match  
need to check whether any node in one space appears in the other space (use hashing)  
need to keep all nodes in a half in memory  $O(b^{d/2})$
- What kind of search in each half space?

## Summary

Criterion	Breadth- First	Uniform- Cost	Depth- First	Depth- Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	$b^{d+1}$	$b^{\lceil C^*/\epsilon \rceil}$	$b^m$	$b^l$	$b^d$
Space	$b^{d+1}$	$b^{\lceil C^*/\epsilon \rceil}$	$bm$	$bl$	$bd$
Optimal?	Yes*	Yes*	No	No	Yes

$b$  branching factor

$d$  solution depth

$m$  maximum depth of tree

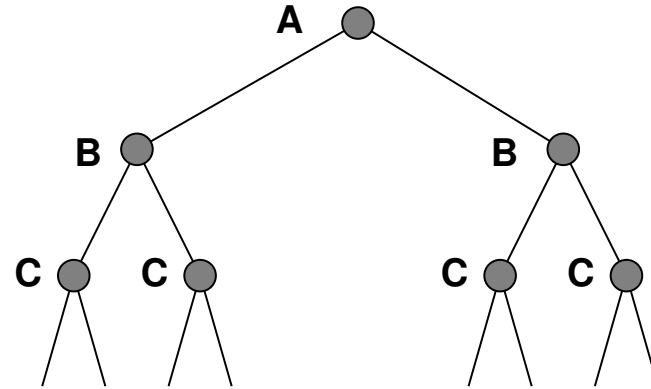
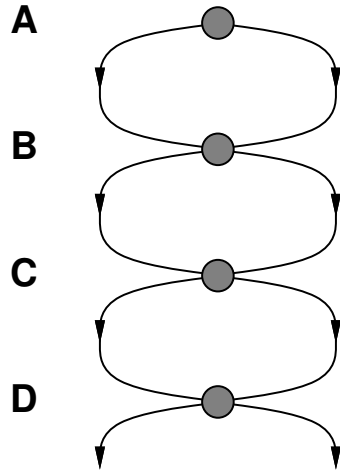
$l$  depth limit

### Loops: Avoid repeated states (I)

Avoid expanding states that have already been visited

Valid for both infinite and finite trees

Example:  $\left\{ \begin{array}{l} m \text{ maximum depth} \\ m + 1 \text{ states} \\ 2^m \text{ possible branches (paths)} \end{array} \right.$

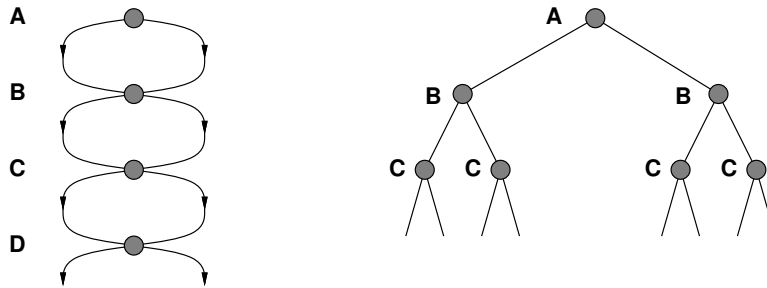


## Loops: (2)

Keep nodes in two lists:  $\left\{ \begin{array}{l} \text{Open list: Fringe} \\ \text{Closed list: Leaf and expanded nodes} \end{array} \right.$

Discard a current node that matches a node in the closed list

Tree-Search  $\longrightarrow$  Graph-Search



Issues:

1. Implementation: hash table, access is constant time  
Trade-off cost of storing+checking vs. cost of searching
2. Losing optimality  
when new path is cheaper/shorter of the one stored
3. DFS and IDS now require exponential storage

## Summary

**Path**: sequence of actions leading from one state to another

**Partial solution**: a path from an initial state to another state

**Search**: develop a sets of partial solutions

- Search tree & its components (node, root, leaves, fringe)
- Data structure for a search node
- Search space vs. state space
- Node expansion, queue order
- Search types: uninformed vs. heuristic
- 6 uninformed search strategies
- 4 criteria for evaluating & comparing search strategies