Homework 7

Assigned on: Friday, November 1^{st} , 2019.

Due: Friday, November 15^{th} , 2019.

Points: 100, plus a potential 20 bonus points in the main tract. Additionally, you have the option of implementing a SAT solver for 100 additional bonus points.

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| 4 | AIMA, Exercise 7.10, page 281. | (16 points) | 1 |
| 5 | Logical Equivalences | (8 points) | 2 |
| 6 | AIMA, Exercise 7.22, page 284. | (18 points + 20 bonus) | 2 |
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| 8 | Bonus: Implementation, Solving SAT | (100 points) | 3 |
| | | | |

Alert: If you submit your homework handwritten, it must be *absolutely neat* or it *will not* be corrected. If you type your homework (preferable), submit using webhandin.

| 1 | AIMA, Exercise 7.1, page 279. | (16 points) |
|----------|-------------------------------|--------------|
| 2 | AIMA, Exercise 7.7, page 281. | (6 points) |
| 3 | Truth Tables | (8 points) |

Use truth tables to show that each of the following is a tautology.

- 1. $(p \land q) \rightarrow \neg(\neg p \lor \neg q)$
- 2. $[Mary \land (Mary \rightarrow Susy)] \rightarrow Susy$

3.
$$\alpha \to [\beta \to (\alpha \land \beta)]$$

4. $(a \to b) \to [(b \to c) \to (a \to c)]$

4 AIMA, Exercise 7.10, page 281.

Only b, c, d, e, f, and g.

(16 points)

5 Logical Equivalences

Using a method of your choice, verify:

- 1. $(\alpha \rightarrow \beta) \equiv (\neg \beta \rightarrow \neg \alpha)$ contraposition
- 2. $\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$ de Morgan
- 3. $(\alpha \land (\beta \lor \gamma)) \equiv ((\alpha \land \gamma) \lor (\alpha \land \beta))$ distributivity of \land over \lor

6 AIMA, Exercise 7.22, page 284. (18 points + 20 bonus)

Parts a, b, and c are required. Parts d, e, and f are bonus.

7 Proofs

Give the explanations of each step if the steps are given, and give both the explanation and step if they are not.

| • If $q \wedge (r \wedge p), t \to v, v \to \neg p$, then $\neg t \wedge r$. | |
|--|--------------|
| Proof | Explanations |
| 1. $q \wedge (r \wedge p)$ | Given |
| 2. $t \rightarrow v$ | Given |
| 3. $v \rightarrow \neg p$ | Given |
| 4. $t \rightarrow \neg p$ | |
| 5. $(r \wedge p)$ | |
| 6. r | |
| 7. p | |
| 8. $\neg \neg p$ | |
| 9. $\neg t$ | |
| 10. $\neg t \wedge r$ | |
| • If $p \to (q \land r), q \to s$, and $r \to t$, then $p \to (s \land t)$. | |
| Proof | Explanations |
| 1. | |
| 2 | |
| 3. | |
| 4. | |
| 5. | |
| 6. | |
| 7. | |
| • Prove by contradiction. | |
| If $\neg(\neg p \land q), p \rightarrow (\neg t \lor r), q$, and t, then r. | |
| Proof | Explanations |
| $1 \neg (\neg n \land a)$ | Given |
| $\begin{array}{ccc} & & & & & \\ & & & & & \\ 2 & & & & & \\ & & & &$ | Given |
| Σ P $(\cdot \cdot \cdot \cdot \cdot)$ | Given |

(8 points)

(28 points)

| 3. q | Given |
|-------------|------------------------|
| 4. <i>t</i> | Given |
| 5. ¬ | Negation of Conclusion |
| 6. | |
| 7. | |
| 8. | |
| 9. | |
| 10. | |
| 11. | |
| 12. | |

8 Bonus: Implementation, Solving SAT (100 points)

Write a search algorithm to determine the satisfiability of a SAT instance. You can either write:

- A DPLL procedure (backtrack search),
- A local search procedure.

You must

- Clearly describe, in addition to your code, your data structures, how your search algorithm operates, and the improvements, if any, that you have included in your code.
- We recommend that you use the standard file for input files known as the 'simplified version of the DIMACS format': http://www.satcompetition.org/2009/format-benchmarks2009.html
- Test the performance of your algorithm on some non trivial uniform random instances taken from the SAT Competition. For example: http://www.cs.ubc.ca/~hoos/SATLIB/benchm.html

Alert: many implementations exist in the literature and on the web. We expect you to do your own implementation.