Week 8 Recitation

Robert Woodward

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- (1 min max) Go over quiz from last week
- (3 min max) Go over homework from last week. Common errors were as follows:
  1. Never use an example as a proof. You may use counter-example only to disprove.
  2. Many students used some variation of the following incorrect notation, which does not mean anything:

     $$A \times B = (a, b), \text{ where } a \in A \text{ and } b \in B.$$

     Correct notation:

     $$a \in A, b \in B, \text{ and } (a, b) \in A \times B.$$

     The first expression is not a well-formed statement, while the second one is.
  3. (Similar to 2.2:8g) If you have $\{a, a\}$, the set is really $\{a\}$ because, in a set, elements are *not* repeated. Therefore, $\{a\} \subset \{a, a\}$ is *false* since $\{a\} \not\subset \{a\}$ (Note that $\{a\} \subseteq \{a\}$ though).
  4. If you are typing in LaTeX always verify that you are using the correct symbols and that displays properly when printed! If you are running over the margin, you may consider to split your expressions into a new line.
  5. Follow the homework guidelines, it specifies that your homework *must* be single sided, whether it is handwritten or printed. Some students still disregard this and other guidelines. Penalties are being applied.
- Questions about lecture / homework so far?
- Rosen 2.3:19, but determine whether f is injective, surjective, bijective, and invertible (If not invertible, what is the largest domain in which f is invertible). f is defined over $\mathcal{R} \to \mathcal{R}$.

  a) $f(x) = 2x + 1$

     1. Injective: $f(x_1) = f(x_2) \Rightarrow 2x_1 + 1 = 2x_2 + 1 \Rightarrow x_1 = x_2$, therefore it is injective.

2. Surjective: For some element $b \in$ rng(f), $b = 2a + 1$, for some element $a \in \mathcal{R}$. Therefore, the range is the set of all reals. So, it is surjective.
3. Bijective: Yes, it is both injective and surjective
4. Invertible: $f^{-1}(y) = x \Rightarrow y = 2x + 1 \Rightarrow \frac{y-1}{2}$

b) $f(x) = x^2 + 1$

1. Injective: $f(x_1) = f(x_2) \Rightarrow x_1^2 + 1 = x_2^2 + 1 \Rightarrow x_1^2 = x_2^2 \Rightarrow \pm x_1 = \pm x_2$, therefore it is not injective.
   Could have also proved this using a proof by counter-example: Consider $f(x) = 4$, there are two pre-images, $x = -2$ and $x = 2$.
2. Surjective: For some element $b \in$ rng(f), $b = a^2 + 1 \Rightarrow \pm\sqrt{b-1} = a$, for some element $a \in \mathcal{R}$. Therefore, the range is the set of all real. So, it is surjective.
3. Bijective: No, it is not injective.
4. Largest domain that it is invertible: $\mathcal{R}^+$

- Let $f(x) = x - 4$ and $g(x) = (x + 1)^2 + 1$. What is $f \circ g$?
  $f \circ g = f(g(x)) = f((x+1)^2 + 1) = (x+1)^2 + 1 - 4 = x^2 + 2x + 1 + 1 - 4 = x^2 + 2x - 4$.
- (Parts from Rosen 8.1:3) Let:
  - $S = \{1, 2, 3, 4\}$
  - $R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$
  - $R_2 = \{(1, 2), (2, 3), (3, 4)\}$

  What is:

  - The 0-1 matrix representation of R_1?
    $$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
  - Is $R_1 \cup R_2$ irreflexive? No, it is reflexive because $\forall i M_{i,i} = 1$
  - Is $R_1 \cup R_2$ transitive? Yes, because $\forall i, j, k; M_{i,j} = 1 \wedge M_{j,k} = 1 \wedge M_{i,k} = 1$.
  - Is $R_1 \cup R_2$ asymmetric? No, because $M_{1,1} = M_{1,1}$
  - Is $R_1 \cup R_2$ antisymmetric? Yes, because $\forall i, j; M_{i,j} \wedge M_{j,i} \rightarrow i = j$
  - The 0-1 matrix representation of R_2?
    $$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- The 0-1 matrix representation of $R_1 \cup R_2$?

  $$N = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Is $R_1 \cup R_2$ irreflexive? No, it is reflexive because $\forall i N_{i,i} = 1$
- Is $R_1 \cup R_2$ transitive? No, because $N_{1,2} = 1 \wedge N_{2,3} = 1$, but $N_{1,3} = 0$.
- Is $R_1 \cup R_2$ antisymmetric? Yes, because $\forall i, j N_{i,j} \wedge N_{j,i} \rightarrow i = j$
- Is $R_1 \cup R_2$ asymmetric? No, because $N_{1,1} = N_{1,1}$
- Is $R_1 \cup R_2$ symmetric? No, because $N_{1,2} \neq N_{2,1}$
- The 0-1 matrix representation of $R_1 \circ R_2$? (Note: This means you do R_2 then R_1)

  $$M_{R_1 \circ R_2} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

• Quiz (Last 15 minutes)