Week 7 Recitation

Robert Woodward

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- Questions about lecture / homework so far?
- Rosen 2.1:31 – Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

  The elements in $A \times B \times C$ are ordered pairs (x, y, z) where $x \in A$, $y \in B$ and $z \in C$.

  The elements in $(A \times B) \times C$ are ordered pairs (w, z) where $w \in A \times B$ and $z \in C$. This means an element in w looks like (x, y), where $x \in A$ and $y \in B$. Therefore, an element in $(A \times B) \times C$ looks like $((x, y), z)$.

  Therefore, the ordered pairs are different.
- (Similar to 2.1:26) Suppose that $A \cup B = \emptyset$, what can you conclude? (Prove formally)

  Answer: we conclude that $(A = \emptyset) \wedge (B = \emptyset)$.

  The proof is by contradiction. Assume $\neg[(A = \emptyset) \wedge (B = \emptyset)] \Rightarrow (A \neq \emptyset)$ or $(B \neq \emptyset)$ $\Rightarrow$ there is at least an element $a \in A$ or at least an element $b \in B$. Without loss of generality, assume it is $a \in A$, then $a \in A \cup B$, but $A \cup B = \emptyset$, contradiction! Therefore, $A = \emptyset$ and $B = \emptyset$.

  Remember that WLOG is the proof by cases. What we did above is a condensed version of the following two cases:

  1. A is not empty and B is empty $\Rightarrow \exists a \in A$ etc.
  2. A is empty and B is not empty: same as previous case but inverting A and B.

  Because the two cases are so similar, we can condense them into one and add the statement WLOG.
- Rosen 2.3:25

  A function needs to be bijective to be invertible.

  Therefore, we need to check if this is one-to-one (injective) and onto (surjective).

  Is it injective? No, $f(x_1) \neq f(x_2) \Rightarrow |x_1| \neq |x_2| \Rightarrow \pm x_1 \neq \pm x_2$

  Now, if the domain is restricted to the set of nonnegative real numbers. Is it injective? Take a number n in the range, what is its preimage? n, therefore it is injective.

Is it surjective? Take any number n in the range, will it be guaranteed to have a preimage? Yes, therefore it is surjective.

Injective: $f(x_1) = f(x_2) \Rightarrow |x_1| = |x_2| \Rightarrow x_1 = x_2$ ($x_1 = x_2$ because the domain is now restricted to nonnegative real numbers). Therefore, it is injective.

Since it is injective and surjective, the function is also bijective, and therefore invertible.

- Rosen 2.2:19

  Proving the statement: $A \setminus B \subseteq A \cap \bar{B}$.
  $\forall x,\ x \in A \setminus B \Rightarrow (x \in A) \wedge (x \notin B) \Rightarrow (x \in A) \wedge x \in \bar{B} \Rightarrow x \in A \cap \bar{B}$.

  Proving the statement: $A \cap \bar{B} \subseteq A \setminus B$.
  $\forall x,\ x \in A \cap \bar{B} \Rightarrow (x \in A) \wedge (x \in \bar{B}) \Rightarrow (x \in A) \wedge (x \notin B) \Rightarrow x \in A \setminus \bar{B}$.

  $(A \setminus B \subseteq A \cap \bar{B}) \wedge (A \cap \bar{B} \subseteq A \setminus B) \Leftrightarrow (A \setminus B = A \cap \bar{B})$. $\square$

- Rosen 2.3:25

  a. $A \cup B \cup C = \{4, 6\}$
  b. $A \cap B \cap C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
  c. $(A \cup B) \cap C = \{4, 5, 7, 10\}$
  d. $(A \cap B) \cup C = \{0, 2, 4, 5, 6, 7, 8, 9, 10\}$

- Rosen 2.3:7b)

  - Domain: $\mathbb{Z}^+$
  - Range: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

- Quiz (Last 10 minutes)