Week 15 Recitation

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- Questions about lecture / homework so far?
- Rosen 2.4, Problem 17(a), Page 162: Compute $\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j)$

$$\begin{aligned}
\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j) &= \sum_{i=1}^{2}(\sum_{j=1}^{3} i + \sum_{j=1}^{3} j) && (1)\\
&= \sum_{i=1}^{2}(i\sum_{j=1}^{3} 1 + \sum_{j=1}^{3} j) && (2)\\
&= \sum_{i=1}^{2}(i \times (3-1+1) + \frac{3(3+1)}{2}) && (3)\\
&= \sum_{i=1}^{2}(3i+6) && (4)\\
&= \sum_{i=1}^{2} i + 6\sum_{i=1}^{2} 1 && (5)\\
&= 3 \times \frac{2(2+1)}{2} + 6(2+1-1) && (6)\\
&= 3 \times 3 + 6 \times 2 && (7)\\
&= 21 && (8)
\end{aligned}$$

  - In (1), we split the terms in the inner summation.
  - In (2), we move i outside the second summation because the index of the summation does not depend on i. It is as if i was a constant.
  - In (3), we compute the two inner summations.
  - In (5), we split the terms in the unique summation left.
  - The rest should be straightforward.

- Show how to do summation in Maple

1. SSH into cse.unl.edu
2. Type in "maple" to launch Maple
3. Type in "sum(*What to Sum*, *Over what range*);". For example, to sum $\sum_{k=1}^{10} k^2$, you would type "sum(kˆ2, k = 1..10);". Another example, to sum $\sum_{k=1}^{n} k^2$, you would type "sum(kˆ2,k = 1..n);".

- Rosen 4.1, Problem 5, Page 280.

  Using the Principle of Mathematical Induction, we will prove that

  $$\forall n \in \mathbb{N}, 1^2 + 3^2 + 5^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3.$$

  We complete the following four steps:

  1. *State the propositional property:*

     $$P(n) : 1^2 + 3^2 + 5^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3.$$

  2. *Basic Step:* We verify that $P(1)$ is true.

     We must form the basic step. First, we can write out what $P(1)$ is.

     $$P(1) : 1^2 + 3^2 = (1+1)(2 \times 1 + 1)(2 \times 1 + 3)/3.$$

     Second, we need to verify that this holds: $1 + 9 = (2)(2+1)(2+3)/3 \Rightarrow 10 = 2 * 3 * 5 \Rightarrow 10 = 10$.

  3. *State the Inductive Hypothesis:* We assume that $P(k)$ holds for all k positive numbers:

     $$\forall k \in \mathbb{N} 1^2 + 3^2 + 5^2 + \ldots + (2k+1)^2 = (k+1)(2k+1)(2k+3)/3.$$

  4. *Inductive Step:* We show that the conditional statement $P(k) \to P(k+1)$ is true for all positive integers k. That is, we prove that $P(k+1)$ must hold assuming that $P(k)$ holds for all positive integers k. Consider the statement $P(k)$:

     $$1^2 + 3^2 + \ldots + (2k+1)^2 = (k+1)(2k+1)(2k+3)/3.$$

     Adding the term $(2k+3)^2$ on both sides of the equality, we get:

     $$\begin{aligned} 1^2 + 3^2 + \ldots + (2k+1)^2 + (2k+3)^3 &= (k+1)(2k+1)(2k+3)/3 + (2k+3)^2 \\ &= (2k+3)[(k+1)(2k+1)/3 + (2k+3)] \\ &= (2k+3)[(2k^2+3k+1)/3 + (6k+9)/3] \\ &= (2k+3)[(2k^2+9k+10)/3] \\ &= (2k+3)(2k+5)(k+2)/3 \\ &= ([k+1]+1)(2[k+1]+1)(2[k+1]+3)/3 \end{aligned}$$

     Therefore, $P(k+1)$ holds, so we have shown that $P(k) \to P(k+1)$. In conclusion, the statement holds by the Principle of Mathematical Induction. $\square$

- Rosen 4.2, Problem 3, page 291.

  Using the Second Principle of Mathematical Induction (or *Strong* Induction), we will prove that:

  > Using just 3-cent stamps and 5-cent stamps, we can form n cents.

  We complete the following four steps:

  1. *State the propositional property:* $P(n)$: n cents can be formed using just 3-cent stamps and 5-cent stamps for $n \geq 8$.
  2. *Basic Step:* We verify that $P(8)$ holds. $P(8)$: 1 5-cent stamp and 1 3-cent stamp.
  3. *State the Inductive Hypothesis:* We assume that $P(j)$ holds for all $8 \leq j \leq k$.
  4. *Inductive Step:* We show that the conditional statement $[P(1) \wedge P(2) \wedge \ldots \wedge P(k)] \rightarrow P(k+1)$.

     By the inductive hypothesis, $P(k-2)$ must hold. Therefore, $P(k+1)$ must hold because it requires the same amount of coins as $P(k-2)$, plus 1 more 3-cent stamp. Or, $P(k+1) : P(k-2)$ and one more 3-cent stamp. Therefore, $P(k+1)$ holds, so we have shown that $[P(1) \wedge P(2) \wedge \ldots \wedge P(k)] \rightarrow P(k+1)$. In conclusion, the statement holds by the Second Principle of Mathematical Induction. $\square$

- (Last 15 minutes) Quiz