Week 13 Recitation

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- Questions about lecture / homework so far?
- Reminder. To prove that $f(n) \in \Delta(g(n))$, we saw three main techniques:
  1. Applying the definition: Specify n_0 and c (or c_1 and c_2).
  2. The limit method: $lim_{n\to\infty} \frac{f(n)}{g(n)}$.
  3. Applying the rule of L'Hôpital in the limit method: $lim_{n\to\infty} \frac{f(n)}{g(n)} = lim_{n\to\infty} \frac{f'(n)}{g'(n)}$.
- (Similar to Rosen 3.2, problem 11) Let $f(n) = 3n^4 + 1$ and $g(n) = \frac{n^4}{2}$. Find a tight bound of the form $f(n) \in \Delta(g(n))$ and prove this bound formally.

  Intuitively we think that $f(n) \in \Theta(g(n))$. Therefore, we need to show:
  1. $f(n) \in \mathcal{O}(g(n))$ and
  2. $f(n) \in \Omega(g(n))$.

  1. *To show* $f(n) \in \mathcal{O}(g(n))$, we need to find:
     - $c_1 \in \mathbb{R}^+$
     - $n_0 \in \mathbb{N}$

     Such that for every positive integer $n \geq n_0$ we have $f(n) \leq c_1 g(n)$.
     $\forall n \geq n_0 = 1$ we have:

     $$3n^4 = 6\frac{n^4}{2} \leq 6\frac{n^4}{2}$$

     Also,

     $$1 \leq n^4 = 2\frac{n^4}{2}$$

     Adding up the above two expressions, we get

     $$3n^4 + 1 \leq 6\frac{n^4}{2} + 2\frac{n^4}{2} = 8\frac{n^4}{2}$$

     Therefore, $f(n) = 3n^4 + 1 \leq 8\frac{n^4}{2}$ $\forall n \geq n_0 = 1$ and $c_1 = 8$. Consequently, $f(n) \in \mathcal{O}(g(n))$.

2. *To show* $f(n) \in \Omega(g(n))$, we need to find:
   - $c_2 \in \mathbb{R}^+$
   - $n_0 \in \mathbb{N}$

   such that for every integer $n \geq n_0$ we have $f(n) \geq c_2 g(n)$.
   For $n_0 \geq 1$, we have:

$$3n^4 \geq n^4 \geq \frac{n^4}{2}$$

   Obviously, we have:

$$1 \geq 0$$

   Summing up the above, $f(n) = 3n^4 + 1 \geq \frac{n^4}{2} + 0 = \frac{n^4}{2}$ $\forall n \geq n_0 = 1$ and $c = 1$. Consequently, $f(n) \in \Omega(g(n))$.

Because $f(n) \in \mathcal{O}(g(n))$ and $f(n) \in \Omega(g(n))$, then $f(n) \in \Theta(g(n))$.

- (Similar to Rosen 2.4, problem 5a) Write the general summation expression of:

  > the sum of the first n numbers that begins with 2 and in which each successive term is 3 more than the preceding term

  First we can write out the sequence to get an idea of what it will look like:

$$\{2, 5, 8, 11, 14, 17, 20, \ldots\}.$$

  This sequence is an *arithmetic progression* and is of the form $a+nd$, where a is the initial term and d is the common difference. Therefore, the *closed form* of the summation, or series, is:

$$\begin{aligned}
\sum_{i=0}^{n-1} 2 + i \times 3 &= \sum_{i=0}^{n-1} 2 + 3i = 2\sum_{i=0}^{n-1} 1 + 3\sum_{i=0}^{n-1} i \\
&= 2((n-1) - 0 + 1) + 3\frac{(n-1)((n-1)+1)}{2} \\
&= 2n + \frac{3}{2}n(n-1) \\
&= \frac{n(3n+1)}{2}
\end{aligned}$$

- Show how to do summation in Maple. Apparently, the license of Maple on cse.unl.edu expired, we are checking on re-instantiating it.

- A few more quick questions about asymptotics.

- (Last 15 minutes) Quiz