Week 13 Recitation

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November 16, 2010

• Questions about lecture / homework so far?

• Reminder. To prove that \( f(n) \in \Delta(g(n)) \), we saw three main techniques:

1. Applying the definition: Specify \( n_0 \) and \( c \) (or \( c_1 \) and \( c_2 \)).
2. The limit method: \( \lim_{n \to \infty} \frac{f(n)}{g(n)} \).
3. Applying the rule of L'Hôpital in the limit method: \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{f'(n)}{g'(n)} \).

• (Similar to Rosen 3.2, problem 11) Let \( f(n) = 3n^4 + 1 \) and \( g(n) = \frac{n^4}{2} \). Find a tight bound of the form \( f(n) \in \Delta(g(n)) \) and prove this bound formally.

Intuitively we think that \( f(n) \in \Theta(g(n)) \). Therefore, we need to show:

1. \( f(n) \in \mathcal{O}(g(n)) \) and
2. \( f(n) \in \Omega(g(n)) \).

1. To show \( f(n) \in \mathcal{O}(g(n)) \), we need to find:
   - \( c \in \mathbb{R}^+ \)
   - \( n_0 \in \mathbb{N} \)

Such that for every positive integer \( n \geq n_0 \) we have \( f(n) \leq c_1 g(n) \).

\( \forall n \geq n_0 = 1 \) we have:

\[
3n^4 = \frac{6n^4}{2} \leq \frac{6n^4}{2}
\]

Also,

\[
1 \leq n^4 = \frac{2n^4}{2}
\]

Adding up the above two expressions, we get

\[
3n^4 + 1 \leq \frac{6n^4}{2} + \frac{2n^4}{2} = \frac{8n^4}{2}
\]

Therefore, \( f(n) = 3n^4 + 1 \leq \frac{8n^4}{2} \forall n \geq n_0 = 1 \) and \( c_1 = 8 \). Consequently, \( f(n) \in \mathcal{O}(g(n)) \).
2. To show \( f(n) \in \Omega(g(n)) \), we need to find:

- \( c_2 \in \mathbb{R^+} \)
- \( n_0 \in \mathbb{N} \)

such that for every integer \( n \geq n_0 \) we have \( f(n) \geq c_2 g(n) \).

For \( n_0 \geq 1 \), we have:

\[
3n^4 \geq n^4 \geq \frac{n^4}{2}
\]

Obviously, we have:

\[
1 \geq 0
\]

Summing up the above, \( f(n) = 3n^4 + 1 \geq \frac{n^4}{2} + 0 = \frac{n^4}{2} \forall n \geq n_0 = 1 \) and \( c = 1 \).

Consequently, \( f(n) \in \Omega(g(n)) \).

Because \( f(n) \in \mathcal{O}(g(n)) \) and \( f(n) \in \Omega(g(n)) \), then \( f(n) \in \Theta(g(n)) \).

• (Similar to Rosen 2.4, problem 5a) Write the general summation expression of:

the sum of the first \( n \) numbers that begins with 2 and in which each successive term is 3 more than the preceding term

First we can write out the sequence to get an idea of what it will look like:

\[
\{2, 5, 8, 11, 14, 17, 20, \ldots\}
\]

This sequence is an arithmetic progression and is of the form \( a + nd \), where \( a \) is the initial term and \( d \) is the common difference. Therefore, the closed form of the summation, or series, is:

\[
\sum_{i=0}^{n-1} 2 + i \times 3 = \sum_{i=0}^{n-1} 2 + 3i = 2 \sum_{i=0}^{n-1} 1 + 3 \sum_{i=0}^{n-1} i
\]

\[
= 2((n - 1) - 0 + 1) + 3\frac{(n - 1)((n - 1) + 1)}{2}
\]

\[
= 2n + \frac{3}{2}n(n - 1)
\]

\[
= \frac{n(3n + 1)}{2}
\]

• Show how to do summation in Maple. Apparently, the license of Maple on cse.unl.edu expired, we are checking on re-instantiating it.

• A few more quick questions about asymptotics.

• (Last 15 minutes) Quiz